Math 582: Foundations of Combinatorics – Graph Theory
Lecturer: Prof. Sara Billey

Problem Set #2

due Wednesday, January 17, 2018

Reading: Download the 5th edition of Diestel's book via the UW Library. Finish reading Chapter 1, start Chapter 2. Think through some of the exercises in Chapter 1. We can discuss any of them that interest you in problem sessions.

Homework Problems: For each of the problems below, explain your answer fully. No credit will be given for a simple numerical answer.

(1) Find the tree whose Prüfer code is $(2, 3, 4, 4, 3, 2, 1, 6, 5)$

(2) Find a formula for the number of labeled trees of order $n > 2$ with degree sequence $(d_1, d_2, \ldots, d_n)$.

(3) How would you define a proper coloring of a hypergraph? In what ways does your definition generalize nice properties of colorings for simple graphs?

(4) Prove that the number of walks from v_i to v_j of length k is a graph is the (i, j)-entry of A^k where A is the adjacency matrix of the graph. Give a combinatorial interpretation for the diagonal entries of A^2.

(5) Prove Prim's algorithm produces a minimal weight spanning tree.

(6) Show that every automorphism of a tree fixes a vertex.

(7) Show that the minor relation defines a partial ordering on any set of pairwise non-isomorphic finite graphs. Is the same true for infinite graphs?