Math 562: Foundations of Combinatorics – Graph Theory
Lecturer: Prof. Sara Billey

Problem Set #1

due Wednesday, January 10, 2018

Reading: Chapter 1 of "Graph Theory" by Diestel. On-line version available on the class web site. Suggested problems: do as many problems from Chapter 1 as possible to solidify the basic vocabulary.

Homework Problems: For each of the problems below, explain your answer fully. No credit will be given for a simple numerical answer.

(1) Determine which of the following sequences are degree sequences for graphs:
  (a) 6,6,5,5,3,2,1,1
  (b) 5,5,4,4,3,3,2,2,1,1
  (c) 4,4,4,4,3,3
  (d) 7,6,5,4,2,1,1,1,1

(2) Prove that no graph with more than one vertex has all degrees different.

(3) Use Kuratowski's theorem to prove the Peterson graph is not planar?

(4) Call a sequence $d = (d_1, d_2, \ldots, d_n)$ of integers *graphic* if there exists a graph G with this degree sequence. Show that d is graphic if and only if the following sequence is graphic

$$d_2 - 1, d_3 - 1, \ldots, d_{d_1+1} - 1, d_{d_1+2}, d_{d_1+3} \ldots, \ldots, d_n.$$

Note, this sequence may not be in decreasing order.

(5) Given a poset $P = (X, <)$, let $C(P)$ be the graph with vertices X and edges $E = \{xy \,|\, x < y \text{ or } y < x\}$. $C(P)$ is called the *comparability graph* of P. Show that every comparability graph and its complement are perfect.

(6) Chapter 1 Exercise 2

(7) Chapter 1 Exercise 20

(8) Bonus: Describe an algorithm to determine if a sequence is graphic. Use the algorithm to verify the number of distinct degree sequences of graphs of order up to 8.