Math 561: Foundations of Combinatorics
Lecturer: Prof. Sara Billey

Problem Set #6

due Wednesday, Nov 6, 2019

Reading: In Chapter 1, read Section 1.10.

Recommended Problems: Play with these problems before reading the solutions: EC1 Chapter 1. Problems 154, 158, 166, 167.

Homework Problems: For each of the problems below, explain your answer fully. No credit will be given for a simple statement of the answer. Each problem is worth 10 points unless otherwise specified.

1. To be determined. This problem will be assigned based on the lecture Friday on "Juggling card sequences"

2. Exercise 155 from Chapter 1 of EC1.

3. Exercise 169 from Chapter 1 of EC1.

4. Let $f(n)$ be the number of length 3 permutations in S_n. Show $f(n)$ is a polynomial for $n \geq 3$. More generally, let $f_k(n)$ be the number of length k permutations in S_n. What can you say about the function $f_k(n)$ for other values of k?

5. How many colored partitions are there of $n = 10$ where each part can be colored by one of 3 colors?

6. Let $A_n(q)$ be the Eulerian polynomial. Give a combinatorial proof showing there exists and expansion
$$A_n(q) = q \sum_{i \geq 0} a_i q^i (1+q)^{n-2i-1}$$
with nonnegative integer coefficients a_i.

7. As a function of n, how many permutations in S_n have descents in positions 1 and 3 only?

8. (Bonus) How many $n \times n$ matrices over $\mathbb{F}_q$ have a square root? Here a matrix M is the square root of N if $N = M^2$.