Math 561: Foundations of Combinatorics
Lecturer: Prof. Sara Billey

Problem Set #2

due Wednesday, October 11, 2017

Reading: In Chapter 1, read Sections 1.3 - 1.5.

Recommended Problems: Give each of these problems careful consideration before reading the solutions: Chapter 1: 3(f and g), 10, 13, 15(c), 19, 23, 24, 57, 58, 60.

Homework Problems: For each of the problems below, explain your answer fully. No credit will be given for a simple statement of the answer. Each problem is worth 10 points unless otherwise specified.

1. If $f(x)$ is the ordinary generating function of the sequence $\{a_n\}_{n\geq 0}$ then express simply, in terms of $f(x)$, the ordinary generating functions for the following sequences:
   (a) $\{\alpha a_n + c\}_{n\geq 0}$, α, c constants.
   (b) $a_0, 0, a_2, 0, a_4, 0, a_6, 0, a_8, 0, \dots$
   (c) $\{a_{n+2} + 3a_{n+1} + a_n\}_{n\geq 0}$
   (d) $0, 0, 1, a_3, a_4, a_5, \dots$
2. Exercise 12 from Chapter 1 of EC1.
3. Exercise 14 from Chapter 1 of EC1.
4. Exercise 17 from Chapter 1 of EC1.
5. Exercise 29 from Chapter 1 of EC1.
6. Exercise 58(a) from Chapter 1 of EC1.
7. Use RSK to give a combinatorial proof that 321-avoiding permutations are Catalan objects.
8. Compute the number of ways to write the permutations 21, 321, 4321 as a minimal product of adjacent transpositions (reduced expressions). Extend this sequence as far as possible and use Sloane's On-line Encyclopedia of Integer sequences to conjecture an explicit formula for the number reduced expressions of the permutation $n, n-1, \dots, 1$. (Bonus: Prove your conjecture.)