HOW TO GET A PH.D. IN MATHEMATICS IN A TIMELY FASHION

SARA C. BILLEY

Mathematics research is fun, engaging, difficult, frustrating, and different than most 9-5 jobs. This article is meant to provide some tips on making the major transition from mathematics student to independent researcher.

Imagine you are a graduate student in a math Ph.D. program and you have just finished your qualifying exams. You are elated, those exams were tough, you studied really hard. Everyone is congratulating you. But, I say "My condolences". You have just passed the last exam you may ever take in your life. Never again will life be so straightforward. Problem sets are very clear. You know the answer comes from the material covered in class that week. You know when you are finished with a problem set. Someone will read it over soon and give you feedback on what you have done next week. Math research, on the other hand, is never finished. You can use all the tools the world has to offer, which is a very large set. Successes come much less frequently.

As you say goodbye to tests and homework, you begin an entirely new career after 20+ years of being a test taking/ problem set producing machine. To obtain your Ph.D. in mathematics your next major hurdle is to create a new theorem, something no one else on earth knows. You will be a world's expert in your little niche. You need to convert all of your skills to this new task. Some people find this transition easy and natural while others spin their wheels trying to figure out what to do next.

Let me make the following analogy with another major life transition. Say you have just moved from a small town in South Dakota to New York City. When you arrive you know nothing about the city so you take a week long bus tour of the city going to the Empire State Building and the Statue of Liberty. After a week of riding the bus around you have seen many beautiful places. But, as you get off the bus, do you really know the city? No. You don't know all of the hidden gems, the neighborhood restaurants, the parks, the people, and you don't know the bad neighborhoods to avoid, how to avoid parking tickets, and who to call when the plumbing breaks. You are free to explore on your own. To begin to learn the ins and outs of NYC, you might try to make a friend. A friend can show you the gems. They can tailor their recommendations to your tastes and interests.

Well, as you finish your qualifying exams in graduate school, you have just gotten off a long bus tour through mathematics. Someone else has been picking all the sites for you. Now you are free to explore on your own. Along the way, it will be very helpful to have a friend along to show you the gems and help you avoid disasters.

Date: April 27, 2020.

The role of "your friend" is played by your advisor. They can help you identify open problems that are interesting to a wide range of other people. They can help you find references. If you are stuck, they can sometimes give you a suggestion that opens a new door. And, they can help you find a job down the road. So finding an advisor is a critical step in your transition.

0.1. **When should you start looking for an advisor?** Right away if you don't already have one. It is never too early to start looking. I will explain why finding an advisor is a top priority for graduate students.

Let's figure out the timeline for graduation. What do you need to do before you graduate? Typically, Ph.D. candidates must

(1) Pass language exams if any.
(2) Advance to Candidacy (1 hour presentation)
    (a) 4 committee members one of which is the advisor (3+1).
    (b) General knowledge of a field.
    (c) A specific thesis topic.
    (d) Some small results.
(3) Thesis Defense (1 hour presentation)
    (a) 4 committee members one of which is the advisor (3+1).
    (b) Command of the known theorems in a field.
    (c) A new theorem.
(4) Getting a Job
    (a) 3-4 recommendation letters.
    (b) Documented commitment to research, teaching, and/or industry.

That's under 10 required hours of your time over the next several years. Of course, you can't fulfill these requirements without doing a lot of preparation, but the actual time spent doing the remaining presentations and exams is very small. One reason some people get stuck transitioning into research is that the requirements for graduation don't really guide you through the process on a day to day basis.

I should make a side comment here. I will write this article with the idea that the reader will be looking for a job in academia since that is what I know. In addition to academia, I enthusiastically encourage mathematicians to go into industry or teach in high school or many other things. The advice here will hopefully help you too since you are still expected to do research for your Ph.D., but details about how many letters you need and internships are not necessarily universally true outside of academia.

Look at the timeline backwards from graduation. About 1 year before graduation you will have to start applying for jobs. That is the way our job application cycle works. Before you apply for a job you need to have proved at least one good theorem. It takes about a year and a half of solid work attacking different problems before you find one that you can prove. People rarely graduate based on the very first problem they try. Before getting started on a particular problem, there is often about a year's

worth of preparation needed; reading papers, taking topics courses, etc. And before all of that, you should pick your advisor so you know which direction to start heading into. So, counting backwards you need to start working on specific problems 2.5 years before graduation and have an advisor approximately 3.5 years before graduation.

0.2. **How do you choose an advisor?** Remember, time is ticking away. You can't graduate until 3.5 years after you chose an advisor. Therefore, picking an advisor is your Number 1 academic priority after passing your qualifying exams.

(1) Make a list of possible advisers.
    (a) What field are you interested in?
    (b) Who specializes in this area?
    (c) What other areas of math are related?
    (d) Which professors work in these areas?
(2) Researching your prospective advisors.
    (a) Take another class with the professors on your list.
    (b) Go to their website and look at their papers. Look them up on MathSciNet, Google Scholar, and the Math ArXiv. Go to the library to get older papers if necessary. Check the dates of their most recent publications.
    (c) Talk directly to the professor. Ask math questions!
    (d) Ask if there are any interesting extensions of this work?
    (e) Talk to their graduate students. Ask them questions like:
        (i) How much time do they give to students?
        (ii) Do they have good problems?
        (iii) Where are their former students now? DO THEY HAVE JOBS?
        (iv) Do they have tenure?

Do's and don'ts: Do talk regularly to your prospective advisor and start working on a project - reading project, coding project, research project. You might need to take the initiative by knocking on their door, asking if they are available to discuss your findings. Build up a working relationship over a couple months before deciding this is a good fit for your advisor. Then ask the professor if they could be your advisor — in person.

Don't be too picky about your choice of field or advisor. You will like any subject once you get to know enough about it. The important thing is to find someone you are happy working with. There is no need to keep shopping around once you have found a good fit. Finding an advisor is not an exact science, it cannot be optimized in a mathematically precise sense.

Don't ask someone to be your advisor by email; this conversation should be done in person. Don't ask someone who doesn't know you to be your advisor. They will most likely say no. When a professor takes on a student, it is a long term responsibility to work with you and write you letters of recommendation for about 10 years. You need the support of your advisor at least until you get tenure.

0.3. **Work hard on research.** After choosing an advisor, the real work begins. Aim to spend 20 hours per week on research. There is something magical about the 20 hour target. Stephen King mentions in his autobiography that he tries to write for 20 hours per week. Also, Susan Solomon a scientist at NOAA (National Oceanic and Atmospheric Administration/ERL) and a member of the National Academy of Science said in a talk once that she aims for 20 hours of research each week. Both these quotes struck me because 20 hours is the working number I aim for in my research. I got the idea from Arun Ram. He was a senior graduate student while I was in my first year at UCSD. He said it takes about 4000 hours of research to get a Ph.D. So if you work 50 weeks per year for 4 years, 20 hours per week will do the job. 20 hours per week means roughly 4 hours per day 5 days per week. That sounds easy enough. But, what counts as research hours? It means really good hours only. I will give some suggestions for how to use the time once I make the rules clear. You can only count hours where you really focused on your research. Computer programming work only counts at half the normal rate because debugging isn't quite the same as deep thought. You can't count any hours in front of the tv. You can't count hours when you spent half the time text messaging with someone. Turn off all notifications on your devices!

Most classes don't count as research hours. Neither does homework. That is a big change from your old career. Let me elaborate on the notion that classes and homework "don't count", because that's a bit subtle.

If a class is particularly close to your research, you may choose to count it. In fact, there is nothing better than taking a class closely related to your research because with each new subject in the class you can compare what you are learning to your thesis problem(s). You will find lots of interesting connections this way. Such classes are rare. Take them when you get the chance.

You may ask what about taking the classes in algebraic topology, functional analysis, statistics, machine learning, optimization, etc that sound so interesting? When will I get another chance to take those classes? If you want to continue to take classes to expand your knowledge, that's great, just keep in mind how you allocate your time. Give yourself permission to skip problem sets and classes as needed to get your research hours done. For example, if some days you have good research momentum, skip class so you can keep working. If there is a visitor in your research area one week, skip a problem set so you have more time to discuss a project that might lead to a collaboration. Don't over commit to too many things. Leave yourself lots of room in the schedule for research so you can regularly find 20 hours per week for research and keep up your teaching or other duties.

Believe it or not, for years I wrote down the start and stop times for every research session in a calendar. I would put down "1:32-2:40, 3:15-4:00, 7:39-9:05" and so on. At the end of the week, I would add up my hours and put a big circle around the total. I was really happy if I got over 22 hours. If I went down to 19, I would make up for it the next week. This sounds very anal, but it actually really helped me prioritize my time. After I had done my 4 hours for the day, I knew I could relax a bit. If

someone came by and invited me to dinner when I had only finished 2.5 hours that day, I would say "no thank you".

I highly recommend you keep track of your research hours for a while until you are in the habit of doing research for many hours per day. If you find you are only spending 10 hours on research, then you know you need to make some major changes to your schedule. You can't gain momentum on a problem in 10 hours per week or less.

What do you do with those hours? Try to make conjectures and prove little lemmas. Little lemmas add up to bigger lemmas and theorems. Don't aim to prove your main theorem in one big step. Do examples. Memorize important vocabulary. Get computer data. Write down your results, and keep them organized in a notebook. Tell your advisor something new every week. Ask your advisor something every week. Always follow up on your advisor's suggestions. Go to seminars to find out where your work fits into the bigger picture. Telling others about your work is the easiest way to get research hours. Talk to the other faculty in your area about your work. Mentors need to be cultivated. Read, read read. Reading math papers takes a lot of time. You can easily spend 1 hour per page on a hard paper; in fact, you can spend days on one page if that is what it takes to understand every line. Ask a lot of questions.

Initially, I actually had to make a conscientious mental switch from my reading/student brain to my creative inquiry brain. Now that I have had lots of practice, I make the switch fluidly. With every paper I read, even with every page I read, I am asking what more can be done with this idea? Try to do this today. Just close your textbook, ask yourself a related question that isn't written, and try to find the answer.

The hardest part of math research is that excellent results don't happen very frequently. It can easily take a couple years to prove a big theorem. Also, you never know if you are looking down the right or wrong path for a proof until you find it. Counting hours can be a way to create more successes in your day or your week even when all of your conjectures break.

My research motto is "Think deeply of simple things" by Arnold Ross. This quote describes research very succinctly. Every idea can be explored further and connected to more things. No question is too trivial to ask. No question has been answered so thoroughly that it is finished. This quote makes me strive to see beyond what is written and to try to say something new.

0.4. **It's your company.** Mathematicians each run a small business. We work in malls called math departments. Your company sells theorems. Your advisor is a free consultant for your company, not your boss, not your employee. Ultimately YOU are responsible for the success of your company.

In particular, if your advisor is not helping you achieve your goals, you can fire them. They can't fire you because you own the company. But, they can say they

don't want to work with you anymore if you treat them badly. Always treat your free consults appropriately. They are human too.

Once you have some theorems, you will need to advertise them. Accept every invitation you get to speak on your work. Treat every talk as a job talk. You never know who will be asked to recommend a good student to a colleague or who will be on an NSF panel reviewing your grant proposal. Prepare carefully. Pick your words carefully. Avoid talking about the parts of your work that you don't really know. I always try to stay at least 2 steps away from things I don't know. That way if someone asks a question, I can answer the first question but decline to go any further from my comfort zone in public.

0.5. **Getting a Job.** As a mathematician, you have many options for your next job. You can

(1) Teach at a college, university, high school, Navy ship, national park, etc. People find jobs here in the US or somewhere far away.
(2) Work in a research lab doing bio-math, astrophysics, computer science, engineering, chemistry, physics, oceanography,...
(3) Work in a government lab like the National Security Agency or the National Institute of Standards.
(4) Work in an industry such as finance, software design, telecommunications, pharmaceuticals, etc. There was a great article in the Notices of the AMS by Cathy O'Neil about moving from academia into finance at D.E. Shaw. She eventually quit that job and started the blog mathbabe.org where she has lots of advice about being a mathematician in different and unexpected places. I highly recommend reading her blog and her book "Weapons of Math Destruction".

If you want to work outside of academia when you finish your Ph.D., consider taking a course or two outside the math department. Apply for a summer internship. Have coffee with friends who have graduated before you.

0.6. **Becoming a Postdoc or Assistant Professor.** I have appended a sampling of job advertisements from EIMS Employment Listings, a service run by the American Mathematical Society. This should give you a flavor of what universities are looking for in new hires. For more information on getting an academic job:

`http://www.ams.org/employment`

Note, that most listings require 3-4 letters of recommendation. Institutions that specialize in teaching require at least one evaluation of your teaching. These places often want to see a strong research agenda as well, so keep track of the important open problems in your field. Institutions that specialize in research will expect to see letters from well known established researchers. Remember to cultivate your mentors/letter writers well before you need them to write the letters.

Good luck with your new company! Work hard! Share your knowledge with others! Let me know if you have comments on this document. It is a work in progress.

Sara Billey, University of Washington

E-mail address: billey@math.washington.edu