

MATH 340A: Homework 7

Due on Gradescope by **August 18th** at 11:59pm.

Problem 1. Let $V = F[x]$, and choose a polynomial

$$p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \in F[x]$$

Let W be the subspace defined by

$$W = \{f(x) \in F[x] \mid p(x) \text{ divides } f(x)\}$$

and let $V' = V/W$.

- (a) Show that (the equivalence classes of) $\{1, x, \dots, x^{n-1}\}$ form a basis for V' .
- (b) Show that the map

$$\begin{aligned} m : V' &\rightarrow V' \\ f(x) &\mapsto xf(x) \end{aligned}$$

is a linear map, and write down a matrix for m in the above basis. This matrix is called the **companion matrix** of $p(x)$, and is the starting point for a canonical form known as **rational canonical form**.

Problem 2. What was one final presentation that you found interesting and why?

Problem 3. What was your favorite topic in this class?