The Fourier series machine

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Covering section 10.3

The Fourier series machine takes as input a function $f(x)$ and outputs its Fourier series, $FS(x)$.

INPUT

Input a function $f(x)$ which is either:

- defined on the interval $[-L, L]$, or
- defined on $\mathbb{R}$ and periodic with period $2L$.

COMPUTE

Compute $a_0, a_1, a_2, \ldots$ (the cosine coefficients) and $b_1, b_2, b_3, \ldots$ (the sine coefficients) from the Euler-Fourier formulas:

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)\,dx, \qquad n = 0, 1, 2, \ldots$$
$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)\,dx, \qquad n = 1, 2, 3, \ldots$$

OUTPUT

Output the function $FS(x)$, which is the Fourier series with the coefficients computed in the previous step:

$$FS(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)\right].$$

The Fourier Convergence Theorem, from section 10.3, says that if $f(x)$ is piecewise-continuous then for all x in the domain of f,

$$FS(x) = \frac{f(x+) + f(x-)}{2},$$

where $f(x+)$ is the limit from the right and $f(x-)$ is the limit from the left.