

The exact conclusion requested in Problem 19-4 in the text is difficult to achieve. Specifically, it's difficult to construct a chart that is global (i.e., covers all of U) and also has an image that's a cube, as required by the definition of a flat chart. (Probably there's some clever change of coordinates one can make initially to achieve a global flat chart at the end, but finding clever changes of coordinates is not the goal here.)

My primary intention in assigning this problem is for you to go through the construction in the proof of the theorem for this example, working in the original, Cartesian (x, y, z) -coordinates. So let's replace the last sentence in the problem by the following.

Let P be the point with coordinates $(x, y, z) = (1, 1, 1)$. Find explicit formulas for coordinates (u, v, w) for a flat chart for D centered at P ; that is, $(u, v, w) = (0, 0, 0)$ at P . Now drop the requirement that the image of the chart be a cube, and use the same formulas to extend the chart as far as possible within the first octant U ; how much of U can you include in the domain of the chart with your coordinate formulas?

In other words, find a chart such that the $w = \text{constant}$ planes are connected integral submanifolds of D on U , and includes as much of U as possible. Depending on how you execute the details of the construction, you may or may not be able to extend the chart to cover all of U .