Errata for Differential Geometry of Curves and Surfaces
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The following list reflects errata found in the text as of August 10, 2011.

Chapter 1
Page 3 line 10: \( t \in [0, 2\pi] \) should be \( t \in [0, 2\pi/\omega] \).
Page 9 line -1: The distance formula in the display equation should be
\[
\sqrt{\|\vec{p} - \vec{b}\|^2 - \frac{(\vec{a} \cdot (\vec{p} - \vec{b}))^2}{\|\vec{a}\|^2} - \frac{\|\vec{a} \times (\vec{p} - \vec{b})\|^2}{\|\vec{a}\|}}
\]
Page 29 line 23: Definition 1.4.4, in the last line “of order 2.” should be “of order at least 2.”
Page 36 line 1: “Given a function \( \kappa_\theta(s) \),” should be “Given a piecewise continuous function \( \kappa_\theta(s) \),” (This is necessary to ensure that \( \kappa_\theta(s) \) is integrable and that \( \cos \theta(s) \) and \( \sin \theta(s) \) are integrable.)
Page 38 line 7: In Problem 1.5.2, \( \vec{x}'(s) = (1, 0) \) should be the initial condition \( \vec{x}'(0) = (1, 0) \).

Chapter 2
Page 41 line 9: In the assumptions on \( \gamma \) we must also assume that \( \gamma \) is positively oriented.
Page 53 line 21: In Section 2.4, we assume that all curves can be parametrized with a parametrization of class \( C^2 \).

Chapter 3
Page 70 line 8: “parametrized curve” should be “parametrized, regular curve.”
Page 80 line 3-5: The interpretation of the sign of torsion is reversed. We have \( \tau(t_0) > 0 \) at \( \vec{x}(t_0) \) when the curve comes up through the osculating plane and we have \( \tau(t_0) < 0 \) at \( \vec{x}(t_0) \) when the curve goes down through the osculating plane. (Figure 3.5 is correct.)
Page 81 line -1: “contact of order 3” should be “contact of order at least 3.”

Chapter 4
Page 88 line 10: In Stokes’s Theorem, we must also assume that \( \gamma \) is oriented according to the right hand rule with respect to the oriented surface \( S \).
Page 91 line -4: \( AB = 2 \sin \left( \frac{AB}{2} \right) \) should be \( AB = 2 \sin \left( \frac{AB}{2} \right) \).
Page 91 line -3: “Let \( \gamma \) be a regular curve” should be “Let \( \gamma \) be a closed regular curve.”
Page 92 line 4: “\( L \leq 2\pi \)” should be “\( L < 2\pi \).”
Page 121 line 5: "$dF_q$ does not have maximal rank" should be "$dF_q$ does not exist or does not have maximal rank".

Page 121 line 8: "$U \in \mathbb{R}^{2n}$ should be "$U \subseteq \mathbb{R}^{2n}$.

Page 123 line 6: "$\{ x \geq 0 \text{ if } y = 0 \}$ should be "$\{ x \geq 0 \text{ and } y = 0 \}$.

Page 124 line 3: "$\{ x \leq 0 \text{ if } z = 0 \}$ should be "$\{ x \leq 0 \text{ and } z = 0 \}$.

Page 131 line 6: $F(U) = U'$ should be $F(U') = U$.

Page 133 line 2: $dF_q$ should be $dF'_q$.

Page 137 line 2: "for any fixed $u_0 \in I$, along any curve $\tilde{X}(u_0, v)_t$" should read "for any fixed $t_0 \in I$, along any curve $\tilde{X}(t_0, u)_t$".

Page 138 line 18: "is a regular curve" should be "is a disjoint union of regular curves".

Chapter 6

Page 160 line 10: "two-dimensional subspace of $\mathbb{R}^{2n}$" should be "two-dimensional subspace of $\mathbb{R}^{3n}$".

Page 169 line 3: $2L_{12}st$ should be $2L_{12}(q,st)$.

Page 169 line 10: $+\tilde{X}_v(v_0)(v - v_0)^2$ should be $+\frac{1}{2} \tilde{X}_v(v_0)(v - v_0)^2$.

Page 174 line 3: The display equation should read:

$$-L_{ij} = \tilde{N}_i \cdot \tilde{X}_j = \left( \sum_{k=1}^{2} a_k^i \tilde{X}_k \right) \cdot \tilde{X}_j,$$

Page 175 line 2,7: The author incorrectly listed the transpose of the matrix of the differential of the Gauss map instead of the matrix itself. Equation (6.25) should read

$$- \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{pmatrix}.$$

Consequently, Equation (6.26), which defines the Weingarten equations should read

$$\begin{pmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{pmatrix} = - \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}^{-1} \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}.$$

Page 179 line 14,15: $L g^{-1}$ in both of those lines should be $g^{-1} L$.

Page 180 line 1: Because of the previous comment, the matrix $(a_i^j)$ is the transpose of what is given in the text.

Page 181 line 5: Equation (6.30) should be

$$\begin{pmatrix} u' \\ -u' \end{pmatrix} \cdot dp_{\nu} \begin{pmatrix} u' \\ v' \end{pmatrix}.$$
Page 187 line 2: The display equation should read:

\[ L_{11} = \frac{-f h'}{\sqrt{f'(v)^2 + h'(v)^2}}, \quad L_{21} = L_{22} = 0, \quad L_{22} = \frac{f'' h' - f' h''}{\sqrt{f'(v)^2 + h'(v)^2}}. \]

Page 187 line 6: The display equation should read:

\[ \kappa_1(u, v) = -\frac{h'(u)}{f(u)\sqrt{f'(u)^2 + h'(u)^2}}, \quad \kappa_2(u, v) = \frac{f''(u) h'(u) - f'(u) h''(u)}{(f'(u)^2 + h'(u)^2)^{3/2}}. \]

Page 189 line -9: “mean curvature of \( S \)” should be “mean curvature (up to sign) of \( S \)”

Page 191 line 10: \(-Lg^{-1}\) should be \(-g^{-1}L\).

Page 192 line 19: The \( L_{ij} \) matrix is off by a sign.

Page 198 line 14: “a curve \( C \)” should be “a simple, closed curve \( C \)”

Page 201 line 9: \( \vec{\beta} \times \vec{w} = (\vec{\beta} \times \vec{w}) \cdot \vec{w}'' / \|\vec{w}\| \) should be \( \|\vec{\beta} \times \vec{w}\| = |(\vec{\beta} \times \vec{w}) \cdot \vec{w}| / \|\vec{w}\| \)

Page 201 line 18: Equation (6.44) should read

\[ (g_{ij}) = \begin{pmatrix} \|\vec{\beta}\|^2 + \vec{u}\vec{v}^2 & \vec{\beta} \cdot \vec{w} \\ \vec{\beta} \cdot \vec{w} & 1 \end{pmatrix} \]

Page 202 line 20-21: “is a cone if and only if \( \vec{\alpha}'(t) = 0 \)” should be “is a cone if \( \vec{\alpha}'(t) = 0 \)”

Page 204 line 11: \( D' \subset U \) should be \( D' \subset U \)

Page 205 line 8: \( \|\vec{X}_u \times \vec{X}_u\| \neq 0 \) should be \( \|\vec{X}_u \times \vec{X}_u\| \neq 0 \)

Page 207 line 10: In Problem 6.6.10, the beginning “Show that” should read “Prove or disprove that”.

Page 207 line 14-15: “parametrized by \( \vec{Z}(u, v) = (1-t)\vec{X}(u, v) + t\vec{Y}(U(v)) \)” should be “parametrized by \( \vec{Z}(u, v) = (1-t)\vec{X}(u, v) + t\vec{Y}(u, v) \)”

Chapter 7

Page 216 line -3: Remove the comma in the display equation.

Page 222 line -3: The display equation should read

\[ a^i_j = -g^{ik} L_{kj}. \]

Page 244 line -9: The display equation for the parametrization of the torus should be

\[ \vec{X}(u, v) = ((b + a \cos v) \cos u, (b + a \cos v) \sin u, a \sin v), \]

Page 244 line -2-1: “Problem 6.5.5” should be “Example 6.5.5.”
Page 299 line 5: “collinear” should be “collinear and pointing in opposite directions”.

Page 300 line 2: “orange” should be “green”

Page 300 line 16: Example 8.1.9 should be Example 8.4.4

Page 303 line 18: “that it consider a” should be “that it considers a”.

Page 310 line 4: “independent” should be “independent”

Page 314 line 5: The display equation should read

$$\int \int_S K \, dS > 0.$$  

Page 314 line 6: “propositions through from 23 definitions” should be “propositions from 23 definitions”

page 315 line -1: “piecewise regular, simple, closed curve.” should be “piecewise regular, simple, positively oriented, closed curve.”