Homework 7, due on Friday, May 12.

Reading: Finish Chapter 5, and read §§6.1-6.2 in Chapter 6.

Practice Problems

After you do A7(b): Notice that from the disjoint union in Problem A7(b) below, it's fairly easy to see that the claims in exercises 6b, 9, 10, and 11cde of §5.3, pp. 252-253, are true.

§5.4, p. 262: 2.

§5.5, p. 274: 3abcd.

Warm-up for A9(b): Give an example of two sets A and B such that

$$\mathcal{P}(A \cup B) \nsubseteq \mathcal{P}(A) \cup \mathcal{P}(B).$$

Hand-In Problems

§5.3, p. 252: 5ac.

§5.4, pp. 262-263: 8, 9. For Exercise 8, also explain why there is a hypothesis that the sets are nonempty by considering what happens when $A = \emptyset$.

§5.5, p. 275: 10.

Problem A7. Let A and B be subsets of some universal set U. Prove each of the following.

(a) *(OPTIONAL)* $A - (A \cap B) = A - B$. (This is practically the same as Proposition 5.11. If you want to use it in (b), prove it as a lemma.)

(b) The set $A \cup B$ is the disjoint union of the three sets $A - B$, $A \cap B$, and $B - A$.

Problem A8. Let A, B, and C be subsets of some universal set U. Give a proof or a counterexample for each of the following. (Remark, not part of the problem: Compare your results here to the first problem in this assignment, §5.3, p. 252: 5ac.)

(a) $(A \cap B) - C = (A - C) \cup (B - C)$.

(b) $(A \cap B) - C = (A - C) \cap (B - C)$.

Problem A9. Let A and B be subsets of some universal set U. Prove each of the following. (For several - but not all - of the implications, proving the contrapositive is the easiest approach. Note the "warm-up" problem in the practice problems above.)

(a) $A \subseteq B$ if and only if $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

(b) $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$ if and only if $A \subseteq B$ or $B \subseteq A$.