Midterm information. Our first midterm is a week from today, on Friday, April 21, during class. Here's some information, more will be posted on the website over this weekend.

There will be no hand-in homework due next week, but there will be reading and practice problems that are preparation for the midterm. At least some of these will be listed by the end of today (Friday 4/14).

The midterm will be closed book, no notes, but a list of axioms, including properties of real numbers, will be provided, as well as any previously proved results that you are allowed to use on the midterm. These will be on the last page of the exam, so that you may detach the page from the test and more easily consult it during the test. Also the axioms will be numbered, so that you can refer to them by number in your answers (in short answers, proof outline, and in complete proofs), which should save you some time during the test.

Read instructions in the exam problems carefully. You may be asked to give a proof outline, which means you should give a table of steps and reasons, but do not have to write it up into a proof. You may be asked for a proof, in which case you do not have to write up an outline first (except to the extent that you need to do so to organize your thinking). A problem may say, "You may use 'algebra' as a justification in this problem." This means you don't have to justify each step of algebra, and may say either 'algebra' or the main step, e.g., 'distributing' or 'multiplying both sides by xy,' as a reason for an algebraic manipulation. *If the problem does not say "You may use 'algebra' …," you do have to name each axiom used*, as in the examples sent in a recent email about homework 3.

There will be a problem for which you have to justify several steps of algebra (similar to the middle part of the first solution for problem 4(c) of §1.2 in the homework 2 solutions). There may be one or more problems that are identical or almost identical to homework problems (including all practice problems).

Information about inequalitites on the back.

Axioms and some basic results for inequalities.

The universal set is the real numbers. That is, all variables a, b, etc., in the axioms and propositions below are real numbers.

We will regard $a < b$ equivalent to $b > a$; that is, one of these may be replaced by the other without comment. Similarly, $a \leq b$ and $b \geq a$ may be treated as equivalent. Substitution in the sense of replacing a quantity by an equal quantity still applies to inequalities as it did to equations; so for instance, if $a < b$ and $a = c$ and $b = d$, then $c < d$, and the same if $<$ is by $\leq$ in both spots.

"Substitution of equals," which means adding the same thing to both sides or multiplying both sides by the same thing, requires a bit of care when extending to inequalities. Here are the corresponding axioms. They are stated for strict inequalities ($<$), but are also true with every $<$ replaced by $\leq$.

- Inequality addition: If $a < b$ and $c = d$, then $a + c < b + d$; and if $a < b$ and $c < d$, then $a + c < b + d$.
- Multiplying an inequality by a positive number preserves the inequality; that is, if $a < b$ and $0 < c$, then $ac < bc$, and also $ca < cb$.
- Multiplying an inequality by a negative number reverses the inequality; that is, if $a < b$ and $c < 0$, then $bc < ac$, and also $cb < ca$.

We can call the last two "Inequality multiplication." (Or if anyone has a better name to suggest, let me know!)

Some useful results.

Corollary of A1(c). For all real numbers a, we have $(-1) \cdot a = -a$.

Proof. In A1(c), replace a by -1 and then change b to a. Then use the multiplicative identity axiom. The result is

$$(-1) \cdot a = -(1 \cdot a) = -a. \qquad \blacksquare$$

Trichotomy Reversal Theorem. If $a > 0$, then $-a < 0$. If $a = 0$, then $-a = 0$. If $a < 0$, then $-a > 0$.

Proof.