Math 300 **Introduction to Mathematical Reasoning** **Autumn 2017**

Citing Reasons in Proofs

In everyday language, a proof is a just convincing argument. But of course, different people will find different things to be convincing. The mathematical community has developed a more restricted concept of the proof. Over a period of centuries, mathematicians have developed standards and conventions for proofs that will produce results they can rely upon. (For a brief history of this process, see the Preface of *Axiomatic Geometry*, by John M. Lee, which will be on reserve for Math 300 at Odegaard.)

A mathematically correct proof of a theorem is a sequence of statements each justified by one or more reasons of these six types:

- a hypothesis in the theorem,
- a definition,
- an axiom,
- a previously proved theorem,
- a previous step in the proof, or
- the laws of logic.

You will see more discussion and many examples of these types of reasons as the course progresses.

Most proofs, however, do not explicitly state the reason for every step. Reasons that will surely be obvious to the intended readership of the proof are usually omitted. This is why the very first of the *Guideline for Writing Mathematical Proofs* in Appendix A of our text is "Know your audience," or we might say, know your reader.

Who is your reader when you are writing a proof for homework or a test in a mathematics course? Your *real* reader is the instructor or grader for the course, and everything needed for the proof is surely obvious to them! In this situation, you need to write for an "Imagined Reader" who knows the material in the course up to this point, but has not yet thought about the theorem you are proving. Thus your Imagined Reader is will change as the course progresses.

In Math 300, the Imagined Reader will change even more than in other mathematics courses. One of our goals in this course is to see how the rules of algebraic computation are proved from a relatively short list of definitions and axioms. So at first, we will imagine a reader who knows only the list of definitions and axioms on the "Axioms for Real Numbers" handout. This means, for instance, that if you want to change $a + b$ to $b + a$, you will have to state that the reason justifying this step is the Commutativity Axiom.

Our Imagined Reader will soon learn more, and we will not need to state reasons when we use commutativity and associativity. There will be a list on the course website indicating which reasons must be stated explicitly for each homework assignment and quiz.

Important Note: The textbook does not start from a base like our "Axioms for Real Numbers" handout, but instead assumes a more knowledgeable reader. So for the first week or so of the course, you will be giving more detailed reasons in your proofs than the examples in the book.