Math 126 Writing Up Problem 2: Throwing Water Balloons, Building Dumpsters and Some Exam Review

DUE WEDNESDAY, MAY 15th

You may submit answers in groups of no more than two. If you work in a group of two, then one person should write up problem 1 and the other should write up problem 2. And you should work out problem 3 together.

PROBLEM 1: Water Balloon Toss

You and a friend have a water balloon that you are going to toss to each other. The goal is to not let the balloon pop. Let d be the distance between you and your friend. Set up a coordinate system with your throwing hand at the origin. Assume the ball is thrown and caught from the same height (which is 0 in this coordinate system). Suppose the only force acting on the balloon is the force due to gravity, so acceleration is $g = 9.8$ m/s^2 downward. This is just a 2D problem. And assume the initial velocity is $\mathbf{v}(0) = \langle a, b\rangle$, for some constants a and b.

1. In terms of a, b and t, find the speed, the tangential and normal components of acceleration and the curvature.
2. Let t_1 be the time at which the balloon is at it's highest point and t_2 be the time when it reaches your friend on the ground. Find formulas for the following quantities in terms of a and b:
   (a) Find t_1 and t_2.
   (b) Make a table that gives the values of the following at both t_1 and t_2: i. x, ii. y, iii. speed, iv. curvatuve, v. tangential component of acceleration, vi. normal component of acceleration.
3. In terms of d, what initial velocity vector will minimize the speed at which the balloon reaches your friend? (Use calculus).
4. After a lot of practicing, and popping many balloon, you figure out that any balloon that reaches the 'catcher' with speed greater than 12 m/sec will pop no matter how good the catcher is. What is the farthest apart you and your friend can stand before the balloon always pops? (Hint: Use the previous part).
5. Dr. Loveless steps in for your friend. He is 5 meters from you. Give the initial speed and initial angle at which you must throw the balloon so that it reaches Dr. Loveless with a speed of exactly 20 m/s. (There are two answers).

PROBLEM 2: Build me a dumpster.

You are asked to design a large metal dumpster and you want to minimize cost. The sides and bottom are rectangles and the top is covered by a plastic lid.

1. The volume must be 200 square feet (you want it to be big enough so that you can throw your math instructor into it).
2. The top will be a plastic lid that costs approximately \$40.00, no matter the dimensions.
3. The sides will be made up of 12-gauge steels sheets that cost \$1.00 per square foot.
4. The bottom is made of a 10-gauge (thicker) steel sheet that cost \$1.25 per square foot.
5. The cost, material and labor, to weld two metal sheets together is \$0.25 per foot.

Give the dimensions that will minimize the total cost to produce the dumpster and what will the cost be? (You will need to use an equation solver after you set up the key equation you need to solve.)

As part of your answer, you will need to find critical points. Use the second derivative test, to verify that your critical point is a local minimum.

PROBLEM 3: Since Math 126 consists of a variety of topics and skills, I think now is a good time to review and make sure we have everything straight. So I'm going to force you to go find the old final exams and work some of the problems as part of this write up. As you do so, I encourage you to look through the other problems in these finals and ask yourself how you think you are doing on keeping up with the variety of topics. Except for the Taylor series questions, you should be able to understand all the other questions on these exams by Exam 2.

The final exam archive is here: http://www.math.washington.edu/∼m126/finals/final.php

Find and complete the following problems:

1. Spring 2005 Problem 4 (Skateboard ramp)
2. Autumn 2005 Problem 10 (Arc Length and Curvature)
3. Winter 2009 Problem 9 (Set up a double integral)
4. Spring 2006 Problem 7 (Velocity and Acceleration)
5. Autumn 2006 Problem 8 (Velocity and Acceleration)