Math 126 Writing Up Problem 1: Magical Flying Things and Parameterized 3D Motion
DUE FRIDAY, APRIL 15th

I: (*A Flying Broom*) You are watching Gary Potter on his magical flying broom. You impose a coordinate system so that the ground is the xy-plane. At precisely noon the sun is directly overhead and you note that the shadow made by the broom on the ground goes from the point P(1,2,0) to the point Q(3,4,0) (P corresponds to the backend of the broom and Q corresponds to the front end). In addition, you know that the broom is tilted upward making an angle of 30 degree with the horizontal, so Gary is flying upward.

1. Find the length of the broom.
2. At noon, assume the front end of the broom is at a height of 25 feet.
   Find the symmetric equations for the path of the front end of the broom.
3. Give the parameterization of the line in terms of arc length (distance traveled) since noon.
4. Assume Gary is traveling at a constant speed of 24 feet/second. Find the parameterization of the motion in terms of t second after noon. How long does it take to reach a height of 500 feet?
5. Assume Gary is at rest at noon, then accelerates at a constant rate of 3 feet/sec^2. Find the parameterization of the motion in terms of time, t, seconds after noon. How long does it take to reach a height of 500 feet?

II: (*A Magic Carpet*) You are watching Aloddin on his magical flying carpet. The carpet is a flat rectangular planar region. As before, you impose a coordinate system so that the ground is the xy-plane. At noon the sun is directly overhead and you take measurements from the carpet's shadow. The four sided shadow has corners at P(2,5,0), Q(5,9,0), R(6,8,0) and S(9,12,0). In addition, you are given that the vector $\langle -5, 2, -7\rangle$ is a normal vector to the carpet.

1. Find the area of the carpet.
2. At noon, assume the lowest point on the carpet is at a height of 50 feet. Consider the direction vector, $\mathbf{v}$, that points in the direction from the highest point on the carpet to the lowest point on the carpet. If the carpet travels on a straight line path in the direction $\mathbf{v}$ at a constant speed of 4 ft/sec, how long after noon will it take for the lowest point on the carpet to touch the ground?

III: (*Jogging on a Surface*) Florrest Gump just feels like running. The enormous expansive area behind Florrest's house is in the shape of the plane $-2x + y + 5z - 5 = 0$. She is standing outside her house on the plane at the point (0,0,1). She runs at a constant speed of 12 ft/sec from her starting point on the plane in such a way that the projection of her tangent vector on the xy-plane is always parallel to the vector $\langle 1, 1, 0\rangle$ (that is Florrest is always "facing in the direction" of $\langle 1, 1, 0\rangle$ relative to the xy-plane). Let $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ be the vector position function for Florrest at time t seconds.

1. Using the equation for the plane, give a relationship between the rates $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dz}{dt}$.
2. Use the fact that the projection of the velocity vector on the xy-plane is parallel to $\langle 1, 1, 0\rangle$ to give a relationship between $\frac{dx}{dt}$and $\frac{dy}{dt}$.
3. Now write down what it means for the speed to be constant and combine this with the facts above to get a formula for $\frac{dz}{dt}$ alone.
4. Solve for the parametric equations $x(t)$, $y(t)$ and $z(t)$.
5. Where will Florrest be located after she has been running for 6 miles?