Unwrapping Wound Shapes on the Forearm

Concept and Usage

This project investigates what the optimal shape of a band-aid should be to most effectively cover a wound of a given shape. Under fewer assumptions, it may have practical relevance in band-aid design. Primarily, its purpose is to introduce a simple application of surface parametrization.

Introduction

The forearm is modeled as a cylindrical surface $S \subset \mathbb{R}^3$, and a wound as a region $W \subset S$. A patch $B \subset \mathbb{R}^2$ corresponds to a band-aid which is meant to completely cover the wound. The following parametrization is used to wrap a flat plane onto the cylinder:

$$f(\theta, h) = (x, y, z) = (\cos(\theta),\ \sin(\theta),\ h)\,,$$

Setting

The cylinder is given by

$$S = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1,\ z \in (a, b)\}.$$

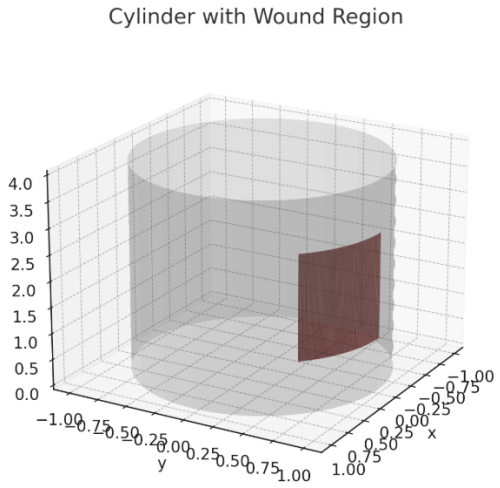
The wound is a circular patch on the cylinder defined by the quadrilateral with corners:

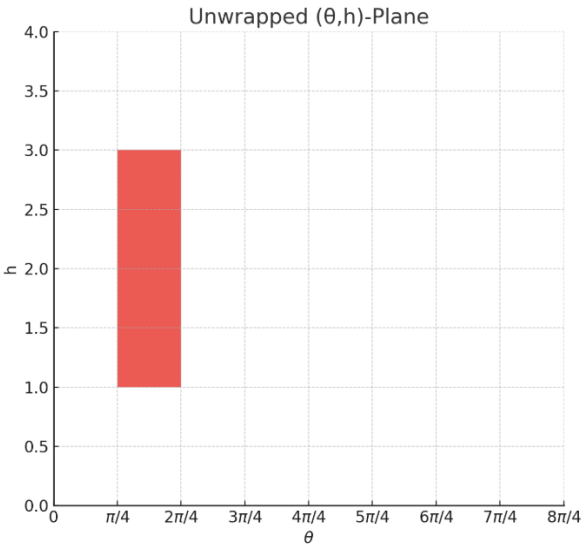
$$\left(\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}, 1\right), \quad (0, 1, 1), \quad \left(\tfrac{\sqrt{2}}{2}, \tfrac{\sqrt{2}}{2}, 3\right), \quad (0, 1, 3).$$

Questions

1. (a) Find the corresponding region $B \subset \mathbb{R}^2$ in the flat (θ, h) plane.
   (b) What are the dimensions of the band-aid patch needed to cover this wound.
2. Confirm that:
   $$ds^2 = dx^2 + dy^2 + dz^2 = d\theta^2 + dh^2.$$
   What does this tell you?

Visuals


Cylinder with Wound Region
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
y
x


Unwrapped (θ,h)-Plane
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
h
0
π/4
2π/4
3π/4
4π/4
5π/4
6π/4
7π/4
8π/4
θ