Weather Maps and Calculus

Katiana Nadolsky

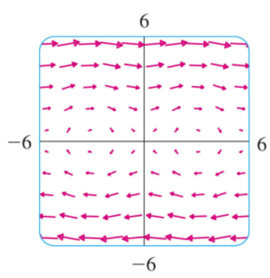
1 Introduction

1.1 Vector Fields

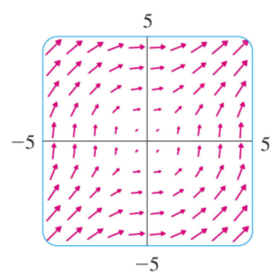
A vector field is a diagram that shows a collection of vectors (arrows modeling magnitude and direction) in different parts of space. These can be a collection of various data points or given in an equation of the form

$$F(x, y) =< u, v >= ui + vj$$

where i is the unit vector in the x-direction and j is the unit vector in the y-direction.



$\mathbf{F}(x, y) = \langle y, \sin x \rangle$



$\mathbf{F}(x, y) = \left\langle \ln\left(1 + y^2\right), \ln\left(1 + y^2\right) \right\rangle$

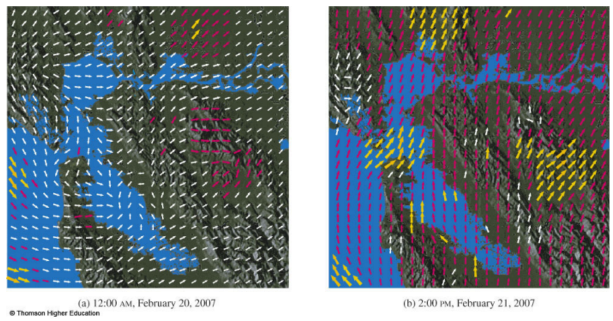
1.2 Weather Charts

A vector field can be used to model wind in a weather chart by measuring its magnitude (wind speed/strength) and direction. We are more focused on the horizontal components of wind, with u as the eastward component and v as the northward component of wind.

Weather charts also provide examples of scalar fields, similar to vector fields, but only a single numerical value is assigned to each space. One example is the magnitude of pressure,

$$p = p(x, y, z)$$

When pressure acts on a surface, it creates a force. While pressure itself is a scalar, the force it exerts on a surface is a vector quantity which we can use to create a corresponding vector field.



Velocity vector fields showing the wind speed and direction

1.3 Gradient

The gradient ∇, which we will use in our later equations,is a way to describe how a function changes at a specific point. It is computed using the partial derivatives of a function and arranging them into a vector. The gradient vector at a point indicates the direction in which the function is increasing most rapidly, and the length of the gradient vector represents the steepness/rate of increase of the function in that direction.

2 Pressure Gradient

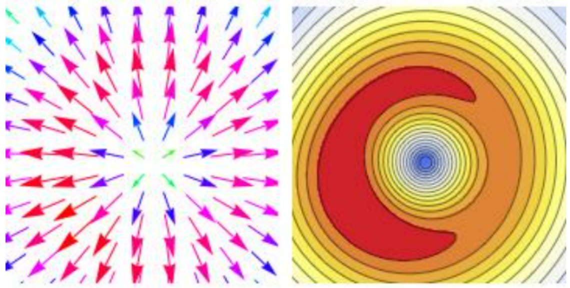
If the pressure is constant, it does not create a force. However, when pressure varies, regions of high pressure push air towards regions of low pressure.

The gradient of pressure is the primary driving force of wind,

$$\nabla P = \frac{dp}{dx}i + \frac{dp}{dy}j$$

where i and j are unit vectors in the x and y directions. This is the horizontal component of the gradient.

Note that wind does not blow following this gradient as the force of pressure is also affected by the force from Earth's rotation, causing it to blow about perpendicular to the pressure gradient.

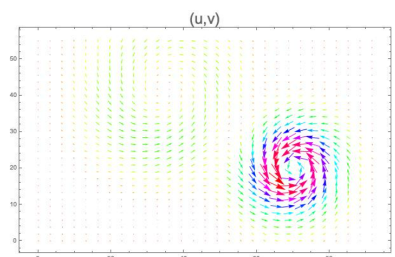


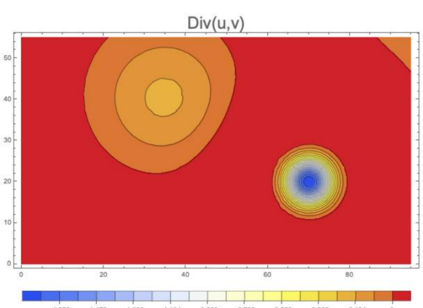
3 Wind Divergence/Convergence

Divergence of the wind vector $F(x, y) =< u, v >$ measures the expansion of air at a given point (net outflow). Calculated by

$$\nabla \cdot F = \frac{du}{dx} + \frac{dv}{dy}$$

Negative divergence, known as convergence, is a measure of contraction.





Divergence is a scalar.

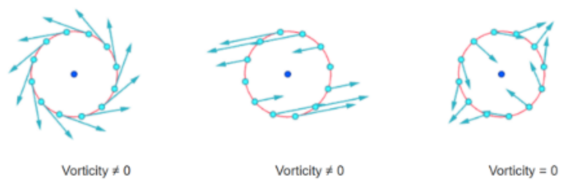
4 Wind Curl/Vorticity

Vorticity, i.e., the curl of the wind vector $F(x, y) =< u, v >$ measures the rotation at a given point. This is calculated by

$$\nabla \times F = \frac{dv}{dx} - \frac{du}{dy}$$

.

It can be thought of as a clockwise or counter-clockwise spin. A positive (increasing) vorticity correlates to a counterclockwise spin, negative (decreasing) vorticity correlates to a clockwise spin, and a vorticity of 0 correlates with no rotation around the center of mass.



While in 2 dimensions curl is a scalar value, note that in 3 dimensions curl is a vector in the z-direction.

5 Pictures

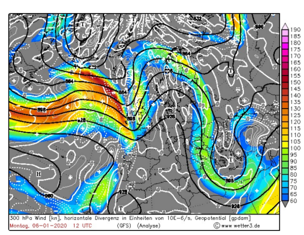


Fig. 6. 300hPa Height (black), Divergence (white) and Windspeed (colours).

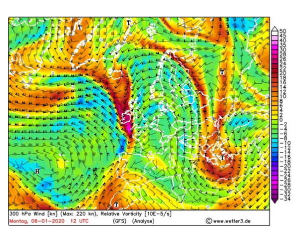


Fig. 7. 300hPa Wind Speed and Direction (arrows) and Relative Vorticity (colours). [Image from \url{http://www1.wetter3.de/}].