Volume of Revolution About $y = mx + b$

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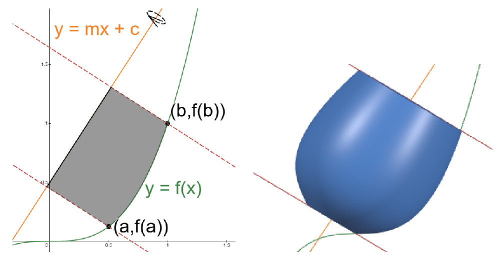
Concepts and usage: Volumes of Revolution. Best used in the last third week of Math 125 at UW.

Introduction For $a \le x \le b$, consider the region bounded by a continuous and differentiable function $y = f(x)$, the line $y = mx + c$, and the perpendiculars that hit the curve at $x = a$ and $x = b$. Under certain conditions the following formula gives the volume of the solid of revolutions around the line.

$$\frac{\pi}{(1+m^2)^{3/2}} \int_a^b (f(x) - mx - c)^2 (1 + mf'(x))\, dx$$



1. **Testing the Formula**: Consider the region below the upper-half circle $f(x) = \sqrt{1-x^2}$, above $y = mx$ and above the perpendicular through the origin for a positive number m. Consider the solid obtained by rotating this region about the line $y = mx$. For all values of m this should give half the volume of a sphere.

   (a) Try $m = 1$: Find the derivative of $f(x) = \sqrt{1-x^2}$ and the intersection of the perpendiculars with the circle (i.e. where does $y = -x$ and $y = x$ intersect the circle), this will give you the values of $x = a$ and $x = b$. Then either compute this integral by hand or enter it into an integrator. What did you get for the volume? Does this match what you expect?

2. **Playing around with the Formula**

   (a) Consider the region(s) bounded by $f(x) = x^2$, $y = mx$ and the perpendiculars to the line that intersect $f(x)$ at $x = 0$ and $x = 1$.

      i. If $m = 1$ what is the volume of the solid obtained by rotating about the line?

      ii. If $m = 0.5$ what is the volume of the solid obtained by rotating about the line?

      iii. What positive value of m minimizes the volume of the solid revolved around $y = mx$?

   (b) Consider the region(s) bounded by $f(x) = x^2$, $y = x + c$ and the perpendiculars to the line that intersect $f(x)$ at $x = 0$ and $x = 1$. What value of c minimizes the volume of the solid revolved around $y = x + c$?

3. **Experiments**

   (a) Can you come up with interesting examples?

   (b) Can you come up with examples were the formula doesn't work?

Visuals to go with the questions above: