Half of a Solid Sphere
by Dr. Andy Loveless

Concepts and usage: Volumes of Revolution. Best used in the last third week of Math 125 at UW.

Introduction: There are an infinite number of ways to 'cut' a solid sphere in half. In this project we will explore such dissections. For all the questions below assume we start with the solid region enclosed by the unit sphere. The first two are doable with volumes of revolution. The others may or may not lead to equations we can solve by hand. Your job is to play around and see how far you can get, but don't be surprised if some get messy, if that happens move on to another one.

1. The Bead Cut: If we drill a cylindrical hole through the center of the sphere, what radius does the cylinder need to be in order to make the volume of the remaining 'bead' equal to half the volume of the sphere.

2. The Double-Cone Cut: At what angle would we have to cut out two identical cone shaped solids out of the top and bottom such the the point of both cones is at the origin and half of the volume remains.

3. Cutting with Planes:

   (a) If you cut the sphere with two parallel vertical planes at $x = -a$ and $x = a$, what is is the value of a that will leave half of the volume remaining?

   (b) If you cut the sphere with the six planes at $x = \pm b$, $y = \pm b$ and $z = \pm b$, what is is the value of b that will leave half of the volume remaining?

4. Each region below is rotated about the y-axis to form a solid. What value of the parameter will make the solid have half the volume of the sphere?

   (a) The region bounded by $x^2 + y^2 = 1$ and $(x - c)^2 + y^2 = 1$, for $0 < c < 1$.

   (b) The region bounded by $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = k$, for $0 < k < 1$.

   (c) The region bounded by $x^2 + y^2 = 1$ and the parabola $x = ay^2$, for $a > 0$.

5. Same regions as above, but now rotate about the x-axis to form a solid. What value of the parameter will make the solid have **one-fourth** the volume of the sphere, so that if we did the same cut on both sides it leave half the volume)?

6. **Make up your own and solve it!**

 Some Visuals to Go With Some of the Examples Above: