Dimples vs. Smooth: The Aerodynamic Advantage

Introduction: When golf balls were first made in the 1800s, they were smooth. Players soon noticed that old, scuffed balls actually flew farther than brand new ones.
That mystery led to the invention of dimples.
Dimples change the airflow around the ball:

- A smooth ball creates a large low-pressure wake behind it → more drag.
- A dimpled ball makes the air turbulent near the surface, shrinking the wake → less drag, more lift.

Today's golf balls all have dimples because they can travel almost twice as far as smooth ones!
Concepts and Usage:
I'm going to try to create a question where I try to simplify finding the distance change with relation to a dimpled vs simple golf ball. Using the equation $dv/dt = -kv$

What k represents:
The constant k is called the drag parameter - it tells us how strongly air resistance affects the ball. Think of it like this:
Higher k = more drag = ball slows down faster
Lower k = less drag = ball maintains speed longer
In reality, air resistance depends on many factors (ball speed, surface texture, air density), but for this problem we're assuming it's simply proportional to velocity.
Why we chose these k values:
Smooth ball ($k = 0.1$): Smooth surfaces create more turbulent wakes behind them, leading to higher drag
Dimpled ball ($k = 0.05$): Dimples create a thin turbulent boundary layer that reduces the size of the wake, cutting drag roughly in half
These values are simplified but realistic, so they capture the key physics that dimples approximately halve the drag coefficient.

Why we avoided the full 2D trajectory equations:(
The complete golf ball problem involves:
2D motion (x and y directions)
Velocity-dependent drag: $F_{drag} = \frac{1}{2}\rho C_d A v^2$
Coupled nonlinear differential equations
No closed-form solution - requires numerical methods

Our simplified 1D model lets us focus on the core insight (how drag affects motion) while keeping the math manageable for a calculus student. We sacrifice some realism for analytical solvability!

Questions:

A golf ball experiences air resistance proportional to its velocity. The equation of motion in the horizontal direction is:

$dv/dt = -kv$

where v is the horizontal velocity and k is the drag constant.

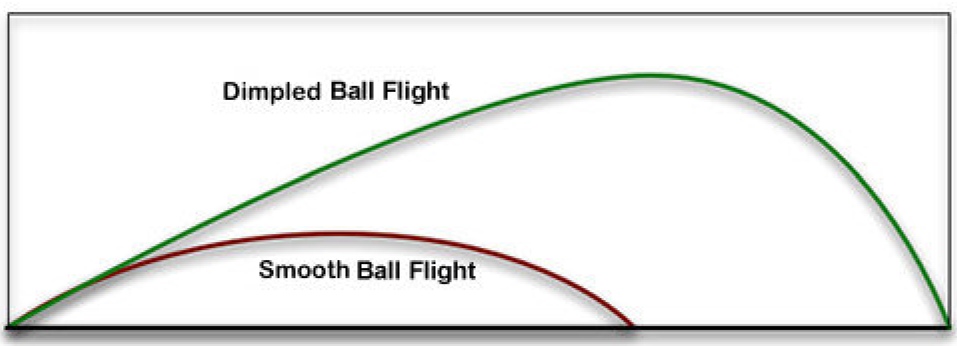
If a smooth ball has $k = 0.1\ s^{-1}$ and a dimpled ball has $k = 0.05\ s^{-1}$, and both start with horizontal velocity $v_0 = 150$ ft/s:

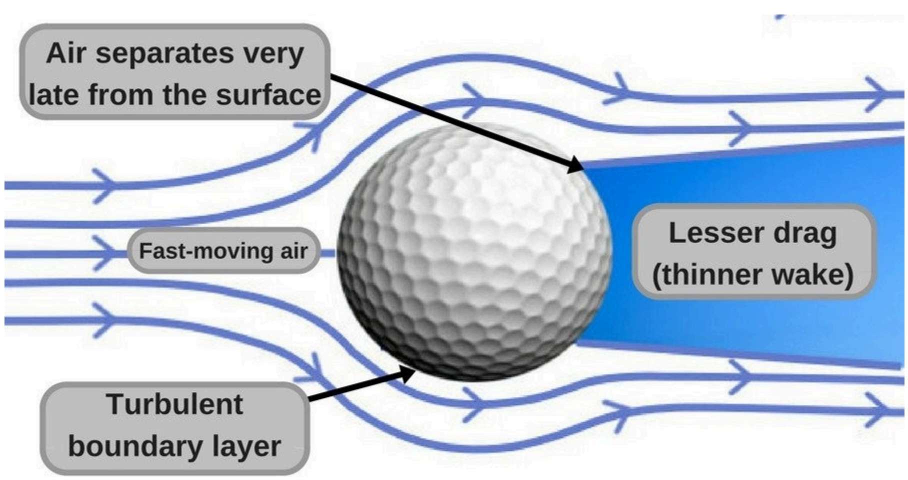
a) Solve the differential equation to find v(t) for each ball

b) Find the horizontal distance traveled after t = 5 seconds

c) How much farther does the dimpled ball travel in those 5 seconds?

Visuals:





Solution

Solution

a) $\frac{dv}{dt} = -kv$

$\frac{dv}{v} = -k\,dt$

$\Rightarrow \ln|v| = -kt + C$

at $t=0$ & $v=150$

$\ln|150| = C$

$\Rightarrow v(t) = 150e^{-kt}$

b) $x(t) = \int v(t)\,dt$

$= \int 150e^{-kt}\,dt$

$= \frac{-150}{k}(e^{-kt}) + C$

at $t=0$ $x=0$ $C = 150/k$

$\Rightarrow x(t) = \frac{150}{k}(1-e^{-kt})$

$\Rightarrow$ at $t=5$

smooth ball $\Rightarrow \frac{150}{0.1}(1-e^{-0.5}) = 591$ ft

dimpled ball $\rightarrow \frac{150}{0.05}(1-e^{-0.25}) = 663$ ft

c) Δ distance $\Rightarrow$ dimpled ball goes 72 ft further.

Sources:
https://math.ucr.edu/home/baez/physics/General/golf.html
https://www.youtube.com/watch?v=fcjaxC-e8oY