Compound Interest and Euler's number e

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Concept and usage: Compound interest formula, limit, natural logarithm and L'Hôpital's rule.

Introduction: Compound interest is interest accumulated from an initial principle and on the accumulated interest from previous periods. Accumulation function can be calculated by the formula: $A = P\left(1+\frac{r}{n}\right)^{tn}$, where A is the final amount, P is the original money being saved, r is the interest rate, n is the compounding frequency, t is the overall length of time the interest is applied (with same time unit as r). When we want to find what \$1 would be able to get after a period, we could use the accumulation function as: $A = \left(1+\frac{r}{n}\right)^{tn}$. Leonhard Euler defined and calculated e to be 2.71828182... with compound interest formula.

Questions:

1. When we have 100% interest rate, one unit of money and save the money for exactly one year. Calculate when we compound the money infinitely often, what is the numerical answer with 3 decimal places. (Hint: Try to use limit to calculate the answer)
2. Calculate lim $\left(1+\frac{r}{n}\right)^{n}$ when n goes to infinity with natural logarithm tricks and L'Hôpital's rule to get a formula.
3. Using the answer from Question 2, find compounding formula $A = P\left(1+\frac{r}{n}\right)^{tn}$ with e.

Visuals: Red line represents original compound formula and green line is continuous compounding formula, where difference can be seen when n is small. If n gets bigger, two lines would be becoming overlap.