Project 1: Exploring Newton's Law of Cooling Via Cup Noodles!

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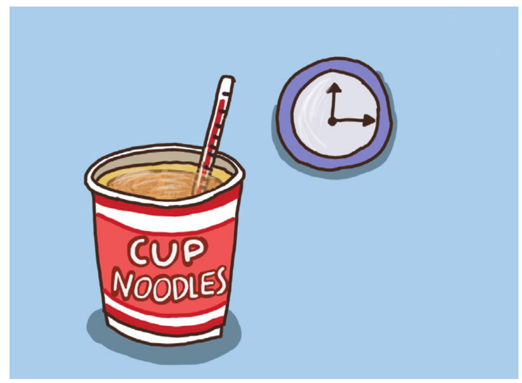
Cup noodles are a popular dinner choice amongst most college students thanks to their convenience. They are known for their quick and easy preparation: requiring an individual to pour hot water into a cup up to a fill line and leaving it to rest for a few short minutes . In this project, we will examine the relationship between time, environmental factors, and how quickly cup noodles cool using Newton's Law of Cooling.



Math Questions:

1. Immediately after uncovering a container of cup noodles in a 78 degree room, they were 180 degrees. Two minutes later the noodles cooled to 175 degrees. Using Newton's Law of Cooling, what is the rate of cooling of the noodles at two minutes?

   $Newton's\ law\ of\ cooling:\ T(t)\ =\ T_s + (T_o - T_s)e^{-kt}$

2. Using linearization, estimate the temperature of the noodles at 2.2 minutes, remember the formula for linearization is: $L(x)\ =\ f(a)\ +\ f'(x)(x\ -\ a)$.

3. Approximate the function centered at a = 2 using a second-order Taylor polynomial
4. A different brand of cup noodles has a measured cooling constant k of 0.03. In the same 78 degree room with a starting temperature of 180 degrees, what is the rate of cooling at two minutes? Assuming both brands use a similar material for their containers, would you expect that this brand to use a thinner or thicker container than the brand from question 1?

Solutions:

(Solutions to 1 & 2 provided by Juno Chen)

1. Newton's Law of Cooling : $T(t) = T_s + (T_0 - T_s)e^{-kt}$

   T_s = temp. of surrounding

   T_0 = initial temp

   - solve for k: $T(2) = 175$

   $$T(2) = 78 + 102e^{-2k} = 175$$
   $$102e^{-2k} = 97$$
   $$\ln e^{-2k} = \ln \frac{97}{102}$$
   $$-2k = \ln\left(\frac{97}{102}\right)$$
   $$k = -\frac{1}{2}\left(\ln\left(\frac{97}{102}\right)\right)$$
   $$k \approx 0.025$$

   - find rate of cooling

   $$T(t) = 78 + (180 - 78)e^{-kt}$$
   $$T(t) = 78 + (102)e^{-kt}$$
   $$\frac{dT}{dt} = 102e^{-kt} \cdot k = -102ke^{-kt}$$
   $$\downarrow$$
   $$\frac{dT}{dt} \text{ at } t = 2 = -102(.025)e^{(-.02505 \cdot 2)}$$
   $$\approx -2.43^\circ F \text{ per minute}$$

2. $$L(x) = f(a) + f'(x)(x-a)$$
   $$f(a) = 175$$
   $$f'(x) = -2.43$$

   $$L(2.2) = 175 - 2.43(2.2 - 2)$$
   $$= 175 - 2.43(.2)$$
   $$= 174.514^\circ F$$

(solution provided by Pisa Chen)

3. Recall that a Taylor polynomial takes the form

   $f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^n(a)}{n!}(x-a)^n$

   where n is the order of the polynomial and a is where the polynomial is centered.

   $T(t) = 78 + (102)e^{-}0.025t$

   $T'(t) = (-0.025)102e^{-}0.025t$

   $T''(t) = (-0.025)^2 102e^{-}0.025t$

   $T(2) = 175$

   $T'(2) = -2.4377$

   $T''(2) = 0.06126$

   Plugging in our values, we get

   $175 - 2.4377(x-2) + \frac{0.06126}{2}(x-2)^2$

(solution provided by Alice Rudders)

4. $\frac{dT}{dt} = (T_0 - T_S)(-k)e^{-kt}$

$\frac{dT}{d(2)} = (180 - 78)(-0.03)e^{-0.03(2)} \approx -2.88$