Chemotaxis and the Gradient

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Concepts and Usage: Partial derivatives and vector magnitudes. Best used in Weeks 5-6 of MATH 126 at the University of Washington.

Introduction: Cells detect and move toward higher concentrations of certain chemicals in a process called chemotaxis. We can model a cell in an environment by treating the chemical concentration as a function and computing its gradient, which this will walk you through in questions. Specifically, we can try to find when the "pull" of two or three chemical sources is equivalent.

Questions:

1. First, we need to figure out what might model the concentration of a chemical in a space. Let's say that the source of a certain chemical is located at (a, b) and the concentration at any other point (x, y) is inversely proportional (with constant k_1) to the distance from the center point squared. Write a function $C_1(x, y)$ representing these facts.

2. Now, let's add in another concentrated point at (c, d). Write a function using the same facts from the previous step for this point, using proportionality constant k_2 and calling the concentration at a point $C_2(x, y)$.

3. Next, we need to figure out which way is optimal for a cell to go to get towards its goal of the chemical source. We can do this by first calculating partial derivatives and then putting them in a vector to represent the direction of the steepest possible ascent, called the gradient vector (I think further math classes cover this more thoroughly, I learned about gradients through AMATH 301 but this definition and explanation should suffice). Calculate the gradient vector $\langle \frac{\partial C_1}{\partial x}, \frac{\partial C_1}{\partial y} \rangle$ and $\langle \frac{\partial C_2}{\partial x}, \frac{\partial C_2}{\partial y} \rangle$.

4. We know know the direction of steepest ascent, but we now need to find how steep it actually is – in other words, how good that "pull" actually is. For this, we can take the magnitude of the gradient vectors. Compute both magnitudes.

5. Finally, we can set these magnitudes equal to each other to solve for the curve where the pull is equivalent. Find and classify this curve. Also consider what happens when $k_1 = k_2$. Is the result the same?

6. **Bonus:** We can do something similar with three points! Create another concentration function, calculate the gradient vector magnitude, and set it equal to the other two gradient vector magnitudes. What type of "curve" is this? Does it exist at all? If it does, do you think it will continue to exist with more and more points, and why?

Visuals: This image actually comes from an application of the answer curve in magnetism!

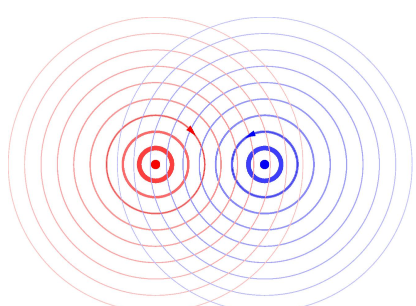

Figure 1: Concentration points with decreasing concentration around them