Angle of Repose and Sand piles

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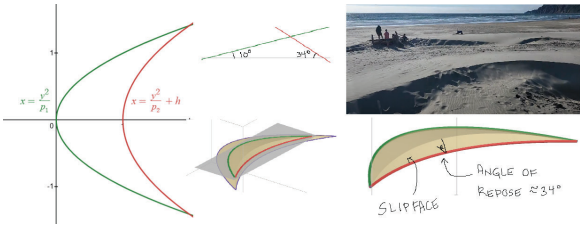
Concepts and usage: Volumes and Surfaces for Math 126.
Introduction: The 'angle of repose' is the steepest angle a granular material can have without slumping. For dry sand the angle of repose is typically given as 34 degrees. We will use this to explore some 3D surfaces.

Project 1: A **Barchan dune** is a type of sand dune formed by wind blowing in one direction. Some research articles model this shape using parabolas. Assume the base of the dune is the region in the xy-plane bounded by $x = \frac{y^2}{p_1}$ and $x = \frac{y^2}{p_2} + h$ for some positive constants p_1, p_2, and h, where $p_2 \geq p_1$. The angle of the sand on the "back" or "slipface" of the dune is 34 degrees as given by the angle of repose. The angle on the front side varies due to the wind speed and how the sand is deposited, but a common angle is around 10 degrees.

Questions: Assume the slipface is 34 degrees and the front side is 10 degrees and all horizontal cross-sections are similar parabolic regions.

- Find the volume and surface area when $p_1 = 1$, $p_2 = 2$ and $h = 1$.
- Can you find a general formula? Can you find a general formula if the front side angle is some other fixed angle α?



Project 2: Sand piles on a support. Assume a platform is made to form a given region and then sand is slowly dumped on that platform until is forms a shape that won't take any more want. Once again assume the angle of repose of dry sand to be 34 degrees.

Questions: Find the volume of the solid formed by the sand if the region is...

- a square? a rectangle? a triangle? a circle? a square with a square hole? a circle with a circular hole? Try other regions, what is the resulting shape and volume?
- Can you make any general statements about the volume based on the region? Is there a relationship between area of the region and the volume?