12.5,12.6, 10.1, and 13.1 Review

This review sheet discusses, in a very basic way, the key concepts from these sections. This review is not meant to be all inclusive, but hopefully it reminds you of some of the basics. Please notify me if you find any typos in this review.

1. **12.5: Lines and Planes in 3 Dimensions**

- *LINES*
  - If
    $\vec{v} = < a, b, c >$ = 'a vector parallel to the line L'
    $\vec{r_0} = < x_0, y_0, z_0 >$ = 'a vector pointing to a location on L when drawn from the origin'
    $\vec{r} = < x, y, z >$ = 'the variable vector'

    then the line L is given by

    $\vec{r} = \vec{r_0} + t\vec{v}$
    $< x, y, z > = < x_0, y_0, z_0 > + t < a, b, c >$ 'the vector equation'

    $x = x_0 + at \ , \ \ y = y_0 + bt \ , \ \ z = z_0 + ct$ 'the parametric equations'

    $$\frac{x - x_0}{a} = \frac{x - y_0}{b} = \frac{z - z_0}{c}$$ 'the symmetric equations'

  - *LINE FACTS*
    (a) Two lines are parallel $\Longleftrightarrow$ the $\vec{v}$ vectors for each line are parallel.
    (b) Two lines intersect $\Longleftrightarrow$ when we can solve the set of equations given by simultaneously setting the equations for each variable equal.
    (c) Two lines are skew $\Longleftrightarrow$ it they are not parallel and they don't intersect.

- *PLANES*
  - If
    $\vec{n} = < a, b, c >$ = 'a normal vector' = 'a vector orthogonal to the plane P'
    $\vec{r_0} = < x_0, y_0, z_0 >$ = 'a vector pointing to a location on P when drawn from the origin'
    $\vec{r} = < x, y, z >$ = 'the variable vector'

    then the plane P is given by
    $\vec{n} \cdot (\vec{r} - \vec{r_0}) = 0$

    $< a, b, c > \cdot (< x, y, z > - < x_0, y_0, z_0 >) = 0$ 'the vector equation'

    $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ 'the scalar equation'

    $ax + by + cz + d = 0$ 'the linear equation'

  - *PLANE FACTS*
    (a) Two planes are parallel $\Longleftrightarrow$ their normal vectors are parallel.
    (b) The shortest distance from a point (x_1, y_1, z_1) and the plane $ax + by + cz + d = 0$ is given by the formula
    $$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$
    To use this formula to find the distance between parallel planes or skew lines please read Examples 9 and 10 in the text.

2. **12.6: Quadric Surfaces -** Be able to recognize the basic shapes of page 836 of your text. Also, you should have some 3D graphing techniques. Here is what we discussed in class:

   (a) *If one or more variables are absent*, then first graph in the appropriate 2D plane and then extend indefinitely in the unrestricted variable.

   (b) *If all variables are present*, then you should try the following:

   - *Traces*: Choose a variable to fix. Then plug in various values for the fixed parameter and draw the resulting graphs in 2D. Putting these together, you can get an idea of the shape.
   - *Table*: See if you function is one in the Table on page 836 of the text.
   - *Plot points*: You should be able to generate your graph by using traces. However, if you are absolutely stuck, then you can plot points by selecting x and y values and computing the z values. Make a table and plot the points in the three dimensional coordinate system.

3. **10.1 and 13.1: Parametric Equations and Vector Functions -** A *parametric equation* is any set of equations that define the variables x, y and z in terms of a separate parameter.

   - *TO GRAPH PARAMETRIC EQUATIONS IN 2D*

     (a) Make a table of values by first picking different values of the parameter. (Usually, this involves picking $t = 0$ and computing x and y. Then picking $t = 1$ and computing x and y. And so forth.)

     (b) Plot the corresponding (x, y) points in the coordinate plane as usual. (Notice that you don't actually plot the parameter t anywhere. If you want you can label the points with the t values you help you remember).

     (c) Put arrows signifying the direction that the curve is going as t is getting larger.

   - *TO ELIMINATE PARAMETERS IN 2D*

     (a) In a limited number of examples, you may be able to combine the parametric equations and as a result get the corresponding equation involving only x and y. To do this, sometimes you can just solve for t and plug your solution in the other equation. Other times you will need to be more clever by using trig identities.

   A *vector-valued function*, or *vector function*, is parametric equation in three dimensions that is written in the form of a vector. In general, they look like:

   $$\vec{r}(t) =< f(t), g(t), h(t) >$$

   - *TO GRAPH OR ELIMINATE PARAMETERS FOR VECTOR FUNCTIONS* Use the same techniques as described for parametric equations above. In three dimensions, it is usually a better idea to try to eliminate parameters first before trying to graph. But remember, you can always plot points!
   - *THE LIMIT FOR VECTOR FUNCTIONS* To find the limit for a vector function, you simply take the limit of three functions separately. We will use this as we step into the land of three variable derivatives.

   There are many ways to find parametric equations or vector functions from a given function in terms of x and y. Perhaps the simplest way is to simply let $x = t$ and solve for what y is in terms of t.