MATH 324A (Autumn 2009)
Practice Final Exam

Student name: ______________________________

Student number: ______________________________

Signature: ______________________________

Do not start working until instructed to do so.

You have 110 minutes.

Please show your work.

Scientific, but not graphing calculators are allowed.

You may use one 8.5 by 11 double-sided sheet of handwritten notes.

Problem 1 (30 points)	
Problem 2 (10 points)	
Problem 3 (10 points)	
Problem 4 (10 points)	
Problem 5 (20 points)	
Problem 6 (20 points)	
Total	

Problem 1 (30 points) Evaluate the following integrals.

(a) $I = \int_C F \cdot ds$, where $F(x, y) = (y, -x)$ and C is the unit circle $x^2 + y^2 = 1$ oriented counter-clockwise.

(b) $I = \int_D xy \, dA$, where $D = \{(x, y) \in \mathbb{R}^2 \colon 0 \leq x \leq 1, 0 \leq y \leq x^2\}$.

(c) $I = \int_S F \cdot dS$, where $F(x, y, z) = (x^2, y^2 z, -1)$ and S is the part of the surface $z = x^2 + y^2$ which lies inside the cylinder $x^2 + y^2 = 1$. Use the upward orientation of S.

(d) $I = \int_0^4 \int_{\frac{1}{2}y}^{\sqrt{y}} x^2 + y^2 \, dx dy.$

(e) $I = \int_L x^2\, ds$, where L is the line segment from $(1, 2, 3)$ to $(4, 5, 6)$.

(f) $I = \int_S y\, dS$, where S is the surface $z = 3x + y^4$ above the region $D = \{(x, y) \in \mathbb{R}^2 \colon x^2 + y^2 \leq 1\}$.

Problem 2 (10 points) Let E be the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a, $b > 0$ are constants.

(a) Write down a parametrization of E.

(b) Use Green's theorem to show that the area bounded by E is πab. (Represent the area as a curve integral.)

Problem 3 (10 points) Consider the vector field $F(x,y) = \big((y+2)\sin(2x), \sin^2(x)\big)$. It is given that F is conservative on $\mathbb{R}^2$.

(a) Find ALL functions $f\colon \mathbb{R}^2 \to \mathbb{R}$ for which $F = \nabla f$.

(b) Evaluate $\int_\gamma F \cdot ds$, where $\gamma(t) = (te^t, t^3)$, $0 \leq t \leq 1$.

Problem 4 (10 points) Let $F(x, y, z) = (x^2y, x^3, y)$ and let C be the intersection of the hyperbolic paraboloid $z = y^2 - x$ and the cylinder $x^2 + y^2 = 1$. Use Stokes' theorem to evaluate the integral $\int_C F \cdot ds$, where C is oriented as counter-clockwise viewed from above.

Problem 5 (20 points) Let $f\colon \mathbb{R}^3 \to \mathbb{R}$ be a smooth function. Show that $\operatorname{curl} \nabla f = 0$. Hint: write $\nabla f = (f_x, f_y, f_z)$ and compute $\operatorname{curl} \nabla f$ from definition.

Problem 6 (20 points) Find the volume of the solid that the cylinder $r = \cos\theta$ cuts out of the unit sphere $x^2 + y^2 + z^2 = 1$.