MATH 324A (Spring 2010)
Midterm

Student name: ______________________________

Student number: ______________________________

Signature: ______________________________

Do not start working until instructed to do so.

You have 50 minutes.

Please show your work.

Scientific, but not graphing calculators are allowed.

You may use one 8.5 by 11 double-sided sheet of handwritten notes.

Problem 1 (20 points)	
Problem 2 (10 points)	
Problem 3 (10 points)	
Problem 4 (10 points)	
Total	

Problem 1 (20 points) Evaluate the following integrals.

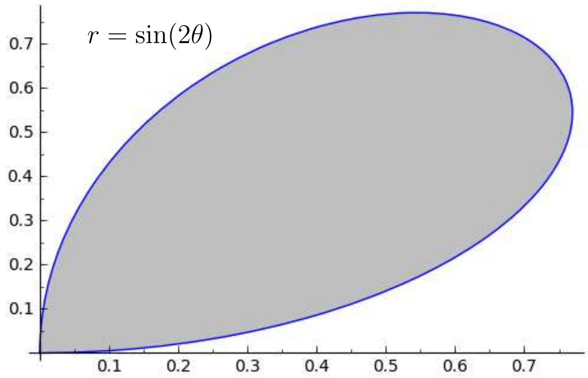
(a) $I = \int_D e^{x^2+y^2}\, dA$, where $D = \{(x,y) \in \mathbb{R}^2 \colon x^2 + y^2 \leq 3\}$.

(b) $I = \int_E x + y + z\, dV$, where $E = \{(x,y,z) \in \mathbb{R}^3 \colon x^2 + y^2 \leq 1,\ x^2 + y^2 \leq z \leq 1\}$. (Think before you compute.)

(c) $I = \int_D e^{x^2}\, dA$, where $D = \{(x, y) \in \mathbb{R}^2 \colon 0 \le y \le 1,\ y \le x \le 1\}$.

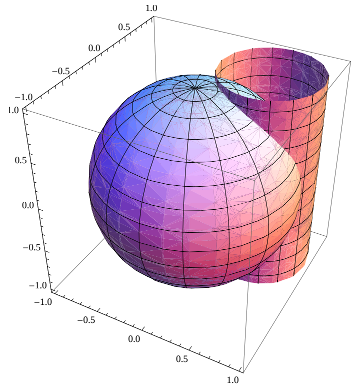
(d) $I = \int_D x - y + 5\, dA$, where $D = \{(x, y) \in [0, 2] \times [0, 2] \colon x + y \le 3\}$. (You may simplify the computation with the fact that the region D has a line of symmetry.)

Problem 2 (10 points) Find the area enclosed by the curve $r = \sin(2\theta)$, $0 \leq \theta \leq \frac{\pi}{2}$.



Problem 3 (10 points) Consider the solid that the cylinder $r = \cos\theta$ cuts out of the unit sphere $x^2 + y^2 + z^2 = 1$.

(a) Setup a triple integral which represents the volume of the solid.



(b) Compute the volume of the solid.

Problem 4 (10 points) Let (X, Y, Z) be a uniformly distributed random point on the unit sphere $\mathbb{S}^2 = \{(x, y, z) \in \mathbb{R}^3 \colon x^2 + y^2 + z^2 = 1\}$. Let (Θ, Φ) be the spherical coordinates of the point, given by

$$\begin{cases} X = \sin(\Phi)\cos(\Theta) \\ Y = \sin(\Phi)\sin(\Theta) \\ Z = \cos(\Phi) \end{cases}$$

You are told that the probability joint density function of (Θ, Φ) is

$$f(\theta, \phi) = \begin{cases} \frac{\sin(\phi)}{4\pi}, & (\theta, \phi) \in [0, 2\pi] \times [0, \pi] \\ 0, & \text{otherwise} \end{cases}$$

What is the probability that $|X| \leq \frac{1}{2}$? (Hint: the sphere $\mathbb{S}^2$ is invariant under rotation.)