# MATH 324A (Autumn 2009)
# Midterm

**Student name:** _____

**Student number:** _____

**Signature:** _____

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Do not start working until instructed to do so.

You have 50 minutes.

Please show your work.

Scientific, but not graphing calculators are allowed.

You may use one 8.5 by 11 double-sided sheet of handwritten notes.

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|---|---|
| Problem 1 (20 points) | |
| Problem 2 (10 points) | |
| Problem 3 (10 points) | |
| Problem 4 (10 points) | |
| Total | |

**Problem 1 (20 points)** Evaluate the following integrals.

(a) $I = \int_D \cos(x^2 + y^2)\, dA$, where $D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 3\}$.

(b) $I = \int_E x^2 + z^2\, dV$, where $E \subseteq \mathbb{R}^3$ is the solid bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 1$. (Hint: use cylindrical coordinates switching the roles of $y$ and $z$.)

(c) $I = \int_D y \, dA$, where $D = \{(x, y) \in \mathbb{R}^2 : (x-1)^2 + y^2 \le 1\}$.

(d) $I = \int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} \, dx \, dy$.

**Problem 2 (10 points)** Find the volume of the solid $E \subseteq \mathbb{R}^3$ which consists of all points satisfying $x^2 + y^2 + z^2 \leq 1$ and $z \geq \sqrt{3(x^2 + y^2)}$. (You are given that $\tan \frac{\pi}{3} = \sqrt{3}$.)

**Problem 3 (10 points)** The joint density function of two random variables $X$ and $Y$ is given by

$$f(x, y) = \begin{cases} C\, e^{-x} e^{-\frac{y}{2}} & x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the constant $C$.

(b) What is the probability of the event $X + Y \leq 1$?

**Problem 4 (10 points)** Use the change of variables

$$\begin{cases} x = u + v \\ y = v \end{cases}$$

to evaluate the double integral $I = \int_R (x - y)^{324} y \, dA$, where $R$ is the parallelogram bounded by the lines $y = x$, $y = x - 1$, $y = 0$ and $y = 1$.