Your Name:

Problem	Points	Possible
1		11
2		10
3		11
4		10
5		8
Total		50

- Turn off and put away cell phones, graphing calculators, books, and notebooks.
- You may use one $8\frac{1}{2} \times 11$ sheet of handwritten notes and a non-graphing calculator. Do not share notes or calculators.
- In order to receive credit, you must **show your work and explain your reasoning**, and give exact answers (unless the problem instructs otherwise). You do not have to simplify answers algebraically.
- Raise your hand if you have a question or need more paper.

Please do not open the test until everyone has a copy and the start of the test is announced.

1. *(11 points total)* Suppose $\iint_D f(x,y)\,dA = \int_0^4 \int_{\sqrt{y}}^2 f(x,y)\,dx\,dy$.

   (a) *(4 points)* Reverse the order of integration in the double integral.

   (b) *(7 points)* Suppose a thin plate with mass density $\rho = ke^y$ occupies the region D, and has total mass m. Find the x-coordinate of the center of mass of the plate. (Your answer will be in terms of k and m.)

2. *(10 points)* A thin plate occupies the region D that lies above the line $y = x$ and inside the circle of radius 2 centered at (2,0). The mass density of the plate is inversely proportional to distance from the origin (that is, the density $= k/r$). Find the total mass of the plate. (Your answer will be in terms of k.)

3. *(11 points total)* Let E be the region in the first octant (that is, $x, y, z \geq 0$) that is inside the sphere $x^2 + y^2 + z^2 = 1$ and below the plane $z = y$.

   (a) *(8 points)* Write $\iiint_E f(x, y, z)\, dV$ as an iterated integral with respect to $dx\, dy\, dz$ (in that order).

   (b) *(3 points)* Explain why it probably would not be a good idea to integrate with respect to z first in the integral in part (a). Make specific use of one or more of the bounding surfaces of E in your explanation.

4. *(10 points)* Set up an integral in spherical coordinates for the volume of the region that is outside the sphere $x^2 + y^2 + z^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 2z$. *Note that you are not asked to evaluate the integral.*

5. *(8 points)* Let R be the region in the xy-plane that is bounded by the ellipse

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1.$$

Use the change of coordinates $2u = x$ and $3v = y$ to evaluate the following integral.

$$\iint_R \left(\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2\right) dA$$