1. (a) Choose \( x \in (A \cup C) - B \). Then \( x \in A \cup C \) but \( x \notin B \). Since \( x \in A \cup C \), \( x \in A \) or \( x \in C \). If \( x \in A \), then we have that \( x \in A \) but \( x \notin B \), which means that \( x \in A - B \), which is a subset of \( (A - B) \cup C \). If \( x \notin A \), then \( x \) must be in \( C \), which means that \( x \in (A - B) \cup C \). Thus, every element of \( (A \cup C) - B \) is also in \( (A - B) \cup C \), which means that \( (A \cup C) - B \subseteq (A - B) \cup C \).

(b) As long as \( B \cap C \) is non-empty, equality will not hold. One example is \( A = \{1, 2, 3\} \), \( B = \{1, 2\} \), and \( C = \{2, 3, 4\} \). Then, \( (A \cup C) - B = \{4\} \) but \( (A - B) \cup C = \{2, 3, 4\} \).

2. (a) The negation is: There exists an \( x \in \mathbb{R} \) such that, for all \( y \in \mathbb{R} \), \( x + y \notin \mathbb{Z} \).

(b) The negation is: For all \( x \in \mathbb{Z} \), there exists \( y \in \mathbb{Z} \) such that \( x > y \) and \( \frac{x^2}{y} \notin \mathbb{N} \).

3. By hypothesis, \( a \) is divisible by \( 3 \). Suppose that \( a + b \) is also divisible by \( 3 \). Then \( a + b = 3k \) for some integer \( k \) and \( a = 3k' \) for some integer \( k' \). So, \( 3k = a + b = 3k' + b \), which implies that \( b = 3(k - k') \). The integers are closed under subtraction, which means that \( k - k' \) is also an integer. Thus, \( b \) is divisible by \( 3 \). We’ve shown that, if \( a + b \) is divisible by \( 3 \), then \( b \) must also be divisible by \( 3 \). Thus, the contrapositive is true: if \( b \) is not divisible by \( 3 \), then \( a + b \) is not divisible by \( 3 \).

4. Base case: If \( n = 1 \), then \( (1 + x)^n = 1 + x \) and \( 1 + 1 \cdot x = 1 + x \) and thus the conclusion is true.

Induction step: Suppose that \( (1 + x)^k \geq 1 + kx \) for some natural number \( k \). We want to show that \( (1 + x)^{k+1} \geq 1 + (k+1)x \). We start with the left-hand side: \( (1 + x)^{k+1} = (1 + x)(1 + x)^k \geq (1 + kx)(1 + x) \), by the induction hypothesis. We now have that \( (1 + x)^{k+1} \geq 1 + kx + x + kx^2 \), which in turn is greater than or equal to \( 1 + kx + x \), since \( kx^2 \geq 0 \). Thus, \( (1 + x)^{k+1} \geq 1 + (k + 1)x \), which completes the induction step.

We’ve shown that \( (1 + x)^n \geq 1 + nx \) for all natural numbers \( n \).

5. \( f(x) \) is injective: Suppose \( f(x_1) = f(x_2) \). Then
\[
\frac{x_1 + 1}{x_1 - 1} = \frac{x_2 + 1}{x_2 - 1}
\]
If we multiply both sides by \( (x_1 - 1)(x_2 - 1) \) to eliminate the denominators, we get:
\[
(x_2 - 1)(x_1 + 1) = (x_2 + 1)(x_1 - 1)
\]
Multiply out and combine like terms to see that \( x_1 = x_2 \).

\( f(x) \) is not onto: There is no \( x \) such that \( f(x) = 1 \). If such an \( x \) did exist, then
\[
\frac{x + 1}{x - 1} = 1
\]
This would mean that \( x + 1 = x - 1 \), which would imply that \( 1 = -1 \). This is a contradiction. Thus, there is an element of the target, \( 1 \in \mathbb{R} \), that is not in the image of \( f \).

Since \( f(x) \) is not onto, \( f(x) \) is not a bijection.