Math 310 HW 3 Hints

1. Problem 3.19: Here is an outline of the proof you need to give. I have left parts out for you to fill in the details.

PROOF We consider 3 cases:

(CASE I): If $x < 0 < y$, then $x^{2k-1} < 0 < y^{2k-1}$ because the power is odd so the number keep the same sign. Thus, this case is done.

(CASE II): If $0 \leq x < y$, then we use induction on $k$.
Base Step: For $k = 1$, we have $x^{2k-1} = x < y = y^{2k-1}$.
Inductive Step: Assume $x^{2k-1} < y^{2k-1}$ for some $k \geq 1$.
We have $x^{2(k+1)-1} = \ldots < \ldots = y^{2(k+1)-1}$
(YOU NEED TO FILL IN THE DOTS. Note that, $x$ and $y$ are both positive, so you can use the axioms of inequalities to say $x^2 < y^2$).

(CASE III): If $x < y \leq 0$, then $0 \leq -y < -x$. So we can use case II with “$x$” replaced by “$-y$” and “$y$” replaced by “$-x$”. (YOU NEED TO FINISH THIS PART AND SHOW THAT $x^{2k-1} < y^{2k-1}$. Induction is not necessary.)

2. Problem 3.26: The recurrence formula for $a_{n+1}$ only requires the previous term $a_n$. So for the base case you only need to show that the formula gives the same value as $a_1$ for $n = 1$. Then you need to make the inductive hypothesis (using basic induction) and use the inductive hypothesis and the recursive formula to prove the formula for $a_{k+1}$.
It is a good idea to expand $(k+1)^3 - (k+1) + 1$ on a scratch sheet of paper before you start so that you can know where you are trying to go.

3. Problem 3.55: The base cases are 1 and 2. You need to show that when you plug 1 into the formula, that you get $a_1$ and when you plug 2 into the formula that you get $a_2$. This is necessary since the recursion is only true for $n > 2$.
After the base case, you must use “strong induction”. This means that you can use the formula for $a(k)$, $a(k-1)$, ..., $a(2)$, $a(1)$ in order to prove that the formula works for $a(k+1)$.
I believe that this is the only place in the homework where “strong induction” is advisable or necessary.