Math 307K, Winter 2012 **Practice Midterm** **University of Washington**

Your Name

Student ID #

- This exam is closed books. No aids are allowed for this exam. You can use any information on the note sheet on the last page without justification.
- In order to receive credit, you must **show all of your work**; to obtain full credit, you must provide mathematical justifications. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 5 pages, plus this cover sheet and the note sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.

Question	Points	Score
1	4	
2	6	
3	4	
4	3	
5	7	
Total	24	

1. (4 points) Find the solution to the initial value problem. Simplify your answer.

$$y' = te^{t+y}, \quad y(0) = 0$$

2. (6 total points)

   (a) (5 points) Find all the solutions to the differential equation

$$y' + 2ty + t = 0$$

   (b) (1 point) Determine an initial condition such that the solution to differential equation does not grow or decay exponentially.

3. (4 total points) Consider the differential equation

$$\frac{dy}{dt} = e^y - 1, \quad -\infty < y_0 < \infty$$

where $y(0) = y_0$.

(a) (2 points) Determine and classify all equilibrium solutions.

(b) (2 points) Determine for which values of y the function is concave and convex, respectively.

4. (3 total points) A mass of 1kg is thrown into the air vertically (i. e. wither upward or downward). The initial velocity and initial position are not specified. The air resistance is measured to be $\gamma|v|$, where v is the velocity of the mass and γ is some positive constant that is to be determined. Take the acceleration due to gravity to be 10m/s^2.

   (a) (2 points) Write down a differential equation that describes the velocity. Your answer should involve γ as a constant.

   (b) (1 point) Suppose it is known that the velocity of the mass will remain constant for the whole duration of the motion if the mass is thrown downward at an initial speed of 100 m/s. Find γ.

5. (7 points) The population of mosquitoes in a certain area increases at a rate proportional to the current population, and in absence of other factors, the population doubles each week. There are 200,000 mosquitoes in the area initially, and predators (birds, bats, and so forth) eat 20,000 mosquitoes/day. Determine the population of mosquitoes in the area at any time.