Math 307 I - Spring 2011
Practice Final
June 03, 2011

Name: ______________________________ Student number: ____________________

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	1	
Total	91	

- Complete all questions.
- You may use a scientific calculator during this examination. Other electronic devices (e.g. cell phones) are not allowed, and should be turned off for the duration of the exam.
- You may use one hand-written 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 120 minutes to complete the exam.

.

1. Find the general solution to the differential equations:

   (a) (5 points)

$$y' = (2 - e^x)/(3 + 2y)$$

   (b) (5 points)

$$ty' + 2y = (\sin\ t)/t.$$

2. Find the general solution to the differential equations:

(a) (5 points)

$$y' = \frac{y^3 - x^3}{2x^3}$$

(b) (5 points)

$$\cos(y)dt + (y^2 - t\sin(y))dy = 0.$$

3. (10 points) Bill is twenty-five years from retirement; in order to retire, Bill needs \$500,000 in his saving's account when he retires in order to maintain his current standard of living. If Bill has \$100,000 in his saving's account right now, and the account earns 5% annually (compounded continuously), how much does Bill need to save each year to reach his goal? (Assume that Bill *continuously* deposits this annual sum into his saving's account.)

4. Solve the following IVP's:

(a) (5 points)

$$y'' - 3y' + 4y = 0 \qquad \begin{cases} y(0) = 1 \\ y'(0) = 0 \end{cases}$$

(b) (5 points)

$$y'' - 3y' + 4y = \sin(t) \qquad \begin{cases} y(0) = 1 \\ y'(0) = 0 \end{cases}$$

5. Find the general solution to the following differential equations:

(a) (5 points)

$$y'' - 4y' + 4y = 0$$

(b) (5 points)

$$y'' - 3y' + 2y = e^{2t}.$$

6. (10 points) Suppose that the motion of a spring-mass system satisfies

$$u'' + u' + 1.5u = \sin(t)$$

and that the mass starts $(t = 0)$ at the equilibrium position from rest. Find the *steady-state solution* (the approximate solution for large values of t).

7. (10 points) Compute the following Laplace transform using the definition (i.e. without using the table):

$$\mathcal{L}\left\{t\, e^{at}\right\}$$

8. (10 points) Find the inverse Laplace transform of

$$F(s) = \frac{e^{-\pi s} - e^{-2\pi s}}{s(s-1)(s-2)}$$

using the table.

9. (10 points) Use the Laplace transform to solve the following IVP:

$$y'' + y = \begin{cases} t/2, & 0 \leq t < 6 \\ t-3, & 6 \leq t \end{cases} \qquad \begin{cases} y(0) = 0 \\ y'(0) = 1. \end{cases}$$

10. (1+ points) Your friend Tim is bad at calculus. He saw you working on the following integral:

$$\int_0^t e^s \cos(s) ds,$$

and suggested that it is equal to

$$\int_0^t e^s ds \cdot \int_0^t \cos(s) ds = (e^t - 1)\sin(t).$$

(a) (1 point) Verify directly that this "solution" is incorrect by computing it's derivative. (Explain what the derivative would be if Tim were correct.)

(b) (2 bonus points) Tim suggests that he was just "unlucky" with $\cos(s)$, and that his rule "usually works." You are not convinced, so you consider Tim's identity:

$$\int_0^t e^s f(s) ds = (e^t - 1) \int_0^t f(s) ds.$$

By differentiating the equation (both sides) twice – and using the Fundamental theorem of calculus – find a separable differential equation that $f(t)$ must satisfy for Tim's rule to work. Solve it for $f(t)$. Does Tim's rule "usually work"?

Table of Laplace transforms:

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \|a\|$
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \|a\|$
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), c > 0$
16. $\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$