

Math 307B - Autumn 2011
Mid-Term Exam 2
February 25, 2011

Name: _____ Student number: _____

1	10	
2	8	
3	10	
4	8	
5	14	
Total	50	

- Complete all questions.
- You may use a calculator during this examination. Other electronic devices are not allowed, and should be turned off for the duration of the exam.
- If you use a trial-and-error or guess-and-check method, or read a numerical solution from a graph on your calculator, when an algebraic method is available, you will not receive credit.
- You may use one hand-written 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 50 minutes to complete the exam.

1. Find the general solution of:

(a)

$$\frac{1}{4}y'' - 2y' + 4y = 0.$$

(5 points)

(b)

$$y'' + 9y = 4 \cos(3t).$$

(5 points)

2. For the following equation, y_1 is a solution. Use the method of reduction of order to find a second solution.

$$t^2 y'' + 2ty' - 2y = 0, \quad t > 0, \quad y_1 = 1/t^2.$$

(8 points)

3. An object of weighing w lbs stretches a spring L feet. It is pulled down x ft and set into motion with initial upward velocity of 3 ft/sec. If the object's motion has frequency 2 sec^{-1} and amplitude 30 in, what is x ? Use $g = 32 \text{ ft/sec}^2$.

(10 points)

4. Suppose a spring system is governed by the equation

$$mu'' + u' + 100u = 0.$$

- (a) The system will be critically damped for which value of m ?
(4 points)

(b) Suppose you can attach one of 3 objects to the above spring:

object	mass
A	1
B	4
C	8

You attach a motor to the system which applies a force of $3 \sin(5t)$ lb. Which of the three objects will give a steady state response of largest amplitude, and why?
(4 points)

5. An object weighing 1 lb stretches a spring 4 ft in a medium which exerts a damping force of $1/2$ lb when the speed of the object is 4 ft/sec. The object is pushed up 3 in and released.

(a) Determine the position of the object at any given time.
(5 points)

(b) When does the object first return to equilibrium position?
(5 points)

- (c) Find a time after which the object remains within $1/10$ in of equilibrium position.
(4 points)

Bonus: Find the general solution of

$$y''' - y = t^2.$$

(2 points)