Chapter 3: Summary of Second Order Solving Methods

We only discussed solution methods for **linear** second order equations.

Constant Coefficient Methods:
To solve an equation of the form: $ay'' + by' + cy = g(t)$.

1. **Homogeneous (when $\mathbf{g(t) = 0}$)**:
   Solve $ar^2 + br + c = 0$.
   (a) If $b^2 - 4ac > 0$, then the general solution looks like $y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$.
   (b) If $b^2 - 4ac = 0$, then the general solution looks like $y(t) = c_1 e^{rt} + c_2 t e^{rt}$.
   (c) If $b^2 - 4ac < 0$, then the general solution looks like $y(t) = c_1 e^{\lambda t}\cos(\omega t) + c_2 e^{\lambda t}\sin(\omega t)$.

2. **Nonhomogeneous (when $\mathbf{g(t) \neq 0}$)**:
   (a) Solve the corresponding homogeneous equation and get independent solutions $y_1(t)$ and $y_2(t)$.
   (b) Find *any* particular solution, $Y(t)$, to $ay'' + by' + cy = g(t)$.
   - Option 1: If $g(t)$ is a product or sum of polynomials, exponentials, sines or cosines, then use **undetermined coefficients**.
   - Option 2: If $g(t)$ involves some function other than those mentioned above, then use **variation of parameters**.

   (c) General Solution: $y(t) = c_1 y_1(t) + c_2 y_2(t) + Y(t)$.

Nonconstant Coefficient Methods:
To solve an equation of the form: $y'' + p(t)y' + q(t)y = g(t)$.

1. **Homogeneous (when $\mathbf{g(t) = 0}$)**:
   (a) If it is an **Euler equation** $(t^2 y'' + \alpha t y' + \beta y = 0)$,
   then you can solve using the transformation from homework.
   For all other situations, one solution MUST be given to you!
   (b) If you are given one solution, $y = y_1(t)$, of $y'' + p(t)y' + q(t)y = 0$,
   then use **reduction of order**.
   That is, write $y(t) = u(t)y_1(t)$, take derivatives and substitute into the equation.
   Then solve for $u(t)$ (you'll have to solve a first order equation using an integrating factor).
   (c) General Solution: $y(t) = c_1 y_1(t) + c_2 y_2(t)$.

2. **Nonhomogeneous (when $\mathbf{g(t) \neq 0}$)**:
   (a) Solve the corresponding homogeneous equation and get independent solutions $y_1(t)$ and $y_2(t)$.
   We can't do this, unless it is an Euler equation, so in most cases you would need to be given independent solutions $y_1(t)$ and $y_2(t)$.
   (b) Find *any* particular solution, $Y(t)$, to $y'' + p(t)y' + q(t)y = g(t)$.
   In this case, there is only one option: use **variation of parameters**.
   (c) General Solution: $y(t) = c_1 y_1(t) + c_2 y_2(t) + Y(t)$