Chapter 2: Summary of First Order Solving Methods

Given $\frac{dy}{dt} = f(t,y)$ with $y(t_0) = y_0$.

1. LINEAR?
   If so, rewrite in the form $\dfrac{dy}{dt} + p(t)y = g(t)$. And use the integrating factor method!

2. SEPARABLE?
   If so, factor, separate and integration: $\dfrac{dy}{dt} = f(t,y) = h(t)g(y) \Longrightarrow \displaystyle\int \frac{1}{g(y)}\,dy = \int h(t)\,dt.$

3. TRY EXACT:
   Rewrite in form $M(t,y) + N(t,y)\dfrac{dy}{dt} = 0$: if $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial t}$, then use the exact method. In other words, start to integrate $\displaystyle\int M(t,y)\,dt + C_1(y)$ and $\displaystyle\int N(t,y)\,dy + C_2(t)$.

4. TRY SUBSTITUTION:
   Let $u =$ 'some expression involving t and y', then simplify this expression and differentiate with respect to t (now you will have $\dfrac{du}{dt}$ in terms of $\dfrac{dy}{dt}$).
   Replace $\dfrac{dy}{dt}$ by $f(t,y)$ in the equation you just found and (using the substitution) try to get a new differential equation that only involves u and t. And HOPE! Hope that the new equation is one you can solve. If I give you such a problem on the test, I will tell you the substitution to use.

Other Notes:

1. If you are asked to find an **explicit** solution, then your final answer needs to be in the form $y = y(x)$. In other words you must solve for y. If you do not solve for y, then we say your answer is an **implicit** solution.

2. Remember to recognize any equilibrium solutions at the beginning. And you can also classify them before you start (this also helps to check your work).

3. Remember to use your initial condition in the end.

4. If $f(t,y)$ is discontinuous or undefined at any t values, then that restricts the domain of our final answer. If $f(t,y)$ or $\dfrac{\partial f}{\partial t}(t,y)$ is discontinuous or undefined at any y, then that also restricts our domain/range of our solution. Solutions do not necessarily exist and are not necessarily unique at or beyond the nearest discontinuity.

5. You can always **check your final answer**! Here is how you check:

   (a) Take your solution and differentiate to find $\frac{dy}{dt}$. Substitute what you just found in for $\frac{dy}{dt}$ in your differential equation. Also replace y by $y(t)$ in your differential equation. If both sides of the differential equation are equal, then you have a solution!

   (b) Also check your initial condition.

   (c) If your function works in the differential equation (makes both sides equal) and if your function satisfies the initial condition, then you will know with certainty that you have a solution!