Partial Derivatives Quick Overview

In Math 307, we sometimes see functiosn of the form $f(x, y)$. This is called a mulivariable function. It gives a third value, let's say z, for each valid value pair of values (x, y) (that is $z = f(x, y)$). In Math 126, you will spend several weeks introducing and studying such functions. In this course, we will have a few occasions where we need to find a rate of change with respect to one of the variables. We will define:

$$\frac{\partial f}{\partial x}(x, y) = f_x(x, y) = \text{'the partial derivative of } f \text{ with respect to } x\text{'}.$$
$$\frac{\partial f}{\partial y}(x, y) = f_y(x, y) = \text{'the partial derivative of } f \text{ with respect to } y\text{'}.$$

For this course, you only need to know how to compute simple partial derivatives of functions of the form $f(x, y)$.

Here is how you compute $\dfrac{\partial f}{\partial x}$:
Treat everything in $f(x, y)$ as a CONSTANT except x (*i.e.* treat y like a constant). Then take the derivative with respect to x.

Here is how you compute $\dfrac{\partial f}{\partial y}$:
Treat everything in $f(x, y)$ as a CONSTANT except y (*i.e.* treat x like a constant). Then take the derivative with respect to y.

A few basic examples:

1. If $f(x, y) = x^3 + 2y^5 + 4$, then the partial derivatives are
$$\frac{\partial f}{\partial x} = 3x^2 \qquad \text{Note: } y \text{ is a constant so the deriv. of } 2y^5 \text{ is zero.}$$
$$\frac{\partial f}{\partial y} = 10y^5 \qquad \text{Note: } x \text{ is a constant so the deriv. of } x^3 \text{ is zero.}$$

2. If $f(x, y) = x^4y^3 + 8x^2y + y^4 + 5x$, then the partial derivatives are
$$\frac{\partial f}{\partial x} = 4x^3y^3 + 16xy + 5 \qquad \text{Note: 8 and } y \text{ are coefficients of } x^2\text{, where } y^4 \text{ is just a constant.}$$
$$\frac{\partial f}{\partial y} = 3x^4y^2 + 8x^2 + 4y^3 \qquad \text{Note: } 8x^2 \text{ is the coefficient of } y \text{ and the deriv. of } y \text{ is 1.}$$

3. If $f(x, y) = \frac{x^2}{y^3} = \frac{1}{y^3}\,x^2 = y^{-3}x^2$, then the partial derivatives are
$$\frac{\partial f}{\partial x} = \frac{2x}{y^3} \qquad \text{Note: No need for quotient rule, only an } x \text{ in the numerator.}$$
$$\frac{\partial f}{\partial y} = -3y^{-4}x^2 \qquad \text{Note: Again, no need for quotient rule, only a } y \text{ in the denominator.}$$

4. If $f(x, y) = (x^2 + y^3)^{10} + \ln(x)$, then the partial derivatives are
$$\frac{\partial f}{\partial x} = 20x(x^2 + y^3)^9 + \frac{1}{x} \qquad \text{Note: We used the chain rule on the first term.}$$
$$\frac{\partial f}{\partial y} = 30y^2(x^2 + y^3)^9 \qquad \text{Note: Chain rule again, and second term has no } y.$$

5. If $f(x, y) = xe^{xy}$, then the partial derivatives are
$$\frac{\partial f}{\partial x} = e^{xy} + xye^{xy} \qquad \text{Note: Product rule, and chain rule in the second term.}$$
$$\frac{\partial f}{\partial y} = x^2e^{xy} \qquad \text{Note: No product rule, but we did need the chain rule.}$$