Final Exam Review

The final is comprehensive.

1. Ch. 1: What is a differential equation? What is a solution (how do you check a solution)?

2. Ch. 2: First Order Equations (equations that only involve first derivatives).
   - 2.1: Integrating Factor Method: Solve $\frac{dy}{dt} + p(t)y = g(t)$ by multiplying by $\mu(t) = e^{\int p(t)dt}$.
   - 2.2: Separable Equations. Rewrite in the form $f(y)dy = g(x)dx$. Integrate both sides!
   - 2.3: Applications! Know the applications and translation tools we saw in class and homework.
   - 2.4: Existence and Uniqueness. Difference between linear and nonlinear.
   - 2.5: Autonomous equations. Know how to find and classify equilibrium solutions.
   - 2.6: Exact equations. Solving $M(x,y) + N(x,y) \frac{dy}{dx} = 0$ when $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.
   - 2.7: Euler’s method to approximate.

3. Ch. 3: Linear Second Order Equations
   - 3.2: Linearity/Fundamental Sets of Solutions (Wronskian)
   - 3.1, 3.3, 3.4: Homogeneous Equations. Solve $ar^2 + br + c = 0$ and give form of solutions.
   - 3.4: Reduction of Order: Write $y(t) = u(t)y_1(t)$, substitute, solve for $u(t)$.
   - 3.5, 3.6: Nonhomogeneous Equations: Homogeneous solution plus a particular solution (undetermined coeff. or variation of parameters).
   - 3.7, 3.8: Mass Spring Systems: $mu'' + \gamma u' + ku = F(t)$.
     - (a) $F(t) = 0$ (No external forcing) and $\gamma = 0$ (No damping): The solution has a constant amplitude. We call $\omega_0 = \sqrt{k/m}$ the natural frequency and $T = \frac{2\pi}{\omega_0}$ the period.
     - (b) $F(t) = 0$ (No external forcing) and $\gamma > 0$ (Damping):
       - If $\gamma = 2\sqrt{k/m}$, then the system is critically damped (no oscillations).
       - If $\gamma < 2\sqrt{k/m}$, then there are oscillations with decreasing amplitudes. We call $\mu = \sqrt{\frac{k}{m} - \frac{\gamma^2}{4m^2}}$ the quasi-frequency and $T = \frac{2\pi}{\mu}$ is the quasi-period.
     - (c) $F(t) = F_0 \cos(\omega t)$ (External forcing wave) and $\gamma = 0$ (No damping): If $\omega = \omega_0$, the solution has unbounded growing amplitudes (resonance). If $\omega \neq \omega_0$, then the solution contains two wave functions of different frequencies.
     - (d) $F(t) = F_0 \cos(\omega t)$ (External forcing wave) and $\gamma > 0$ (Damping): The homogeneous solution is called the transient solution (it dies out). The particular solution (the part that remains after the transient solution dies out) is called the steady state response.

4. Ch. 6: The Laplace Transform - (still about Linear Second Order Equations)
   - 6.1: Computing Laplace Transforms (for our basic function and piecewise functions)
   - 6.2: Solving with Laplace Transforms (derivative rules, solving for $L\{y\}$, partial fractions, and inverse Laplace)
   - 6.3: Step Functions (writing discontinuous jumps in terms of step functions).
   - 6.4: Solving discontinuous forcing problems with Laplace transforms.