Final Exam Review

The final is comprehensive.

1. Ch. 1: What is a differential equation? What is a solution (how do you check a solution)?

2. Ch. 2: First Order Equations (equations that only involve first derivatives).
   - 2.1: Integrating Factor Method: Solve $\frac{dy}{dt} + p(t)y = g(t)$ by multiplying by $\mu(t) = e^{\int p(t)\,dt}$.
   - 2.2: Separable Equations. Rewrite in the form $f(y)dy = g(x)dx$. Integrate both sides!
   - 2.3: Applications! Know the applications and translation tools we saw in class and homework.
   - 2.4: Existence and Uniqueness. Difference between linear and nonlinear.
   - 2.5: Autonomous equations. Know how to find and classify equilibrium solutions.
   - 2.6: Exact equations. Solving $M(x, y) + N(x, y)\frac{dy}{dx} = 0$ when $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.
   - 2.7: Euler's method to approximate.

3. Ch. 3: Linear Second Order Equations
   - 3.2: Linearity/Fundamental Sets of Solutions (Wronskian)
   - 3.1, 3.3, 3.4: Homogeneous Equations. Solve $ar^2 + br + c = 0$ and give form of solutions.
   - 3.4: Reduction of Order: Write $y(t) = u(t)y_1(t)$, substitute, solve for $u(t)$.
   - 3.5, 3.6: Nonhomogeneous Equations: Homogeneous solution plus a particular solution (undetermined coeff. or variation of parameters).
   - 3.7, 3.8: Mass Spring Systems: $mu'' + \gamma u' + ku = F(t)$.
     (a) $F(t) = 0$ (No external forcing) and $\gamma = 0$ (No damping): The solution has a constant amplitude. We call $\omega_0 = \sqrt{k/m}$ the **natural frequency** and $T = \frac{2\pi}{\omega_0}$ the **period**.
     (b) $F(t) = 0$ (No external forcing) and $\gamma > 0$ (Damping):
       - If $\gamma = 2\sqrt{k/m}$, then the system is **critically damped** (no oscillations).
       - If $\gamma < 2\sqrt{k/m}$, then there are oscillations with decreasing amplitudes. We call $\mu = \sqrt{\frac{k}{m} - \frac{\gamma^2}{4m^2}}$ the **quasi-frequency** and $T = \frac{2\pi}{\mu}$ is the **quasi-period**.

     (c) $F(t) = F_0 \cos(\omega t)$ (External forcing wave) and $\gamma = 0$ (No damping): If $\omega = \omega_0$, the solution has unbounded growing amplitudes (resonance). If $\omega \neq \omega_0$, then the solution contains two wave functions of different frequencies.
     (d) $F(t) = F_0 \cos(\omega t)$ (External forcing wave) and $\gamma > 0$ (Damping): The homogeneous solution is called the **transient solution** (it dies out). The particular solution (the part that remains after the transient solution dies out) is called the **steady state response**.

4. Ch. 6: The Laplace Transform - (still about Linear Second Order Equations)
   - 6.1: Computing Laplace Transforms (for our basic function and piecewise functions)
   - 6.2: Solving with Laplace Transforms (derivative rules, solving for $\mathcal{L}\{y\}$, partial fractions, and inverse Laplace)
   - 6.3: Step Functions (writing discontinuous jumps in terms of step functions).
   - 6.4: Solving discontinuous forcing problems with Laplace transforms.