Chapter 1 Review: Introduction

Essential Facts from Calculus 1 and 2:

1. $\frac{dy}{dt}$ = 'instantaneous rate of change of y with respect to t' = 'slope of the tangent line to $y(t)$ at t'.
   - $\frac{dy}{dt} = 0 \implies y(t)$ has a horizontal tangent.
   - $\frac{dy}{dt} > 0 \implies y(t)$ is increasing.
   - $\frac{dy}{dt} < 0 \implies y(t)$ is decreasing.
   - Any value of t where $\frac{dy}{dt} = 0$ is called a critical number for $y(t)$ (we should know how to determine if a critical number corresponds to local max, local min, or neither)
2. Integration!! You must know your integration well. You will regularly use substitution and by parts throughout the quarter (and you will use partial fractions frequently in the last two weeks of the quarter).

Application Notes:

1. Translation tools:
   - 'rate of change of BLAH' $\iff \frac{d(BLAH)}{dt}$
   - '...is proportional to...' $\iff$ '...is a constant multiple of...'
   - units of $\frac{dy}{dt} \iff y$-units/t-units
2. Specific Examples:
   - "The rate of change of y is a constant c" $\iff \dfrac{dy}{dt} = c$ (Such as: Deposit an average of $c = 2000$ dollars into an account each year or 500 people immigrate into a city each year...)
   - "The rate of change of y is proportional to y" $\iff \dfrac{dy}{dt} = ky$ (Continuously compounded interest, population growth)
   - "The rate of change of temperature an object is proportional to the difference between the object and the surrounding temperature." $\iff \dfrac{dT}{dt} = k(T - T_s)$
   - Mixing Problems $\iff \dfrac{dy}{dt} =$ (Rate IN) − (Rate OUT) (Note: Rate OUT will involve y in some say and watch your units!)
   - Force Diagram (Motion) Problems $\iff m\frac{dv}{dt} = F$ (F is the sum of all the forces, v is velocity, m is mass).

Classification

1. An **ordinary differential equation** (ODE) is an equation involving derivatives relating two variables (a dependent and independent variable).

2. A **partial differential equation** (PDE) is an equation involving partial derivatives relating to more than two variables (typically, one dependent and two or three independent variables)

3. The **order** of a differential equation is the highest derivative that appears in the equation.

4. A differential equation is said to be **linear** if it only involves a linear combination of first powers of the function y and its derivatives. In other words, a linear differential equation looks like $a_n(t)(t)y^{(n)} + a_{n-1}(t)y^{(n-1)} + \cdots a_1(t)y' + a_0(t)y = g(t)$.
   Here are two examples of linear differential equations:
   $$t^2y' + e^ty = \sin(t) \quad \text{and} \quad 8y'' - ty' + (t^2 - \ln(t))y = e^{-t}.$$

5. We say a differential equation is **nonlinear** if it is not linear (meaning it involves some other power or function of y).
   Here are two examples on nonlinear differential questions:
   $$y' + y^2 = t^3 \quad \text{and} \quad y' + \sqrt{y} = e^y + t^2.$$

6. A linear differential equation is said to have **constant coefficients** if it only has constants (meaning no t's) in front of y and the derivatives of y.
   Here are two examples:
   $$5y' + 10y = t^2 \quad \text{and} \quad 3y'' - 2y' + 10y = \cos(t).$$

7. A linear differential equation is said to be **homogenous** if the function $g(t)$ is always 0.
   Here are two examples:
   $$5y' + 10y = 0 \quad \text{and} \quad y' + t^2y = 0.$$

Slope Fields

1. Since $\frac{dy}{dt}$ is the slope of the tangent line, we can visualize a differential equation by plugging in variables values of t and y and getting slopes (make a table). Then we can plot these slopes by drawing short line segment with the given slope. If we do this at regularly space points, then the resulting graph is called a **slope field**.

2. Facts about a slope field:

   - There will be a horizontal tangent at any (t, y) points where $\frac{dy}{dt}$ is equal to zero.
   - If there is a constant value $y = c$ which satisfies the differential equation (which means the derivative is always zero), then we say $y(t) = c$ is an **equilibrium solution**.
   - Once we know where $\frac{dy}{dt}$ is equal to zero, we can look on either side and determine if $\frac{dy}{dt}$ is positive (slopes upward) or $\frac{dy}{dt}$ is negative (slopes downward).

3. Thus, we can learn a lot of information by studying a differential equation directly. And a slope field is a useful way to depict this information.