Graphs of sol'ns to

$$u'' + 2u' + 5u = 10\cos(t)$$

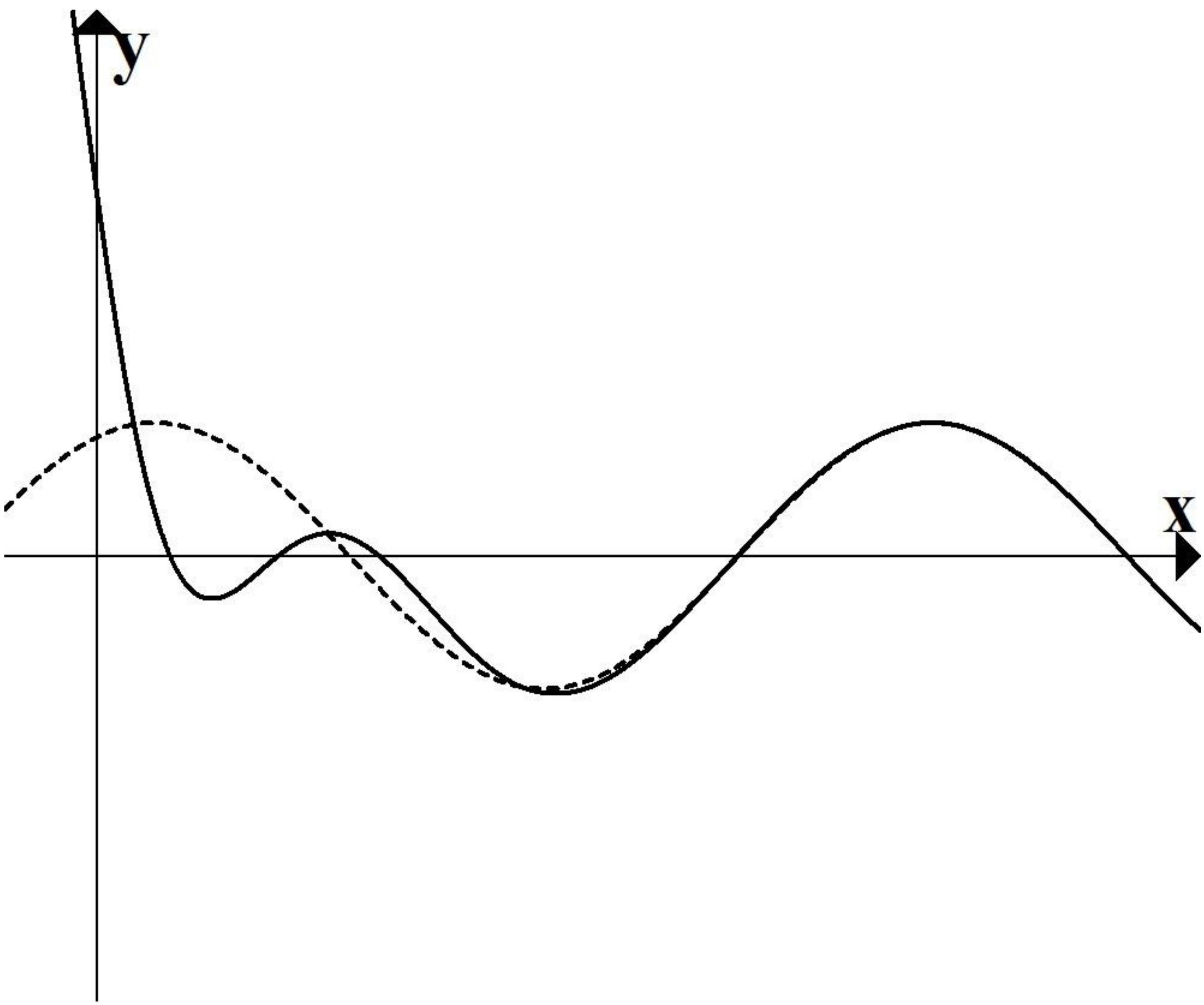
Steady state solution (dotted):

$$U(t) = 2\cos(t) + \sin(t)$$

The solution with initial conditions

u(0) = 6 and u'(0) = -11 is shown. This solution is:

$$u(t) = e^{-t}(4\cos(2\text{t}) - 6\sin(2t)) + U(t)$$

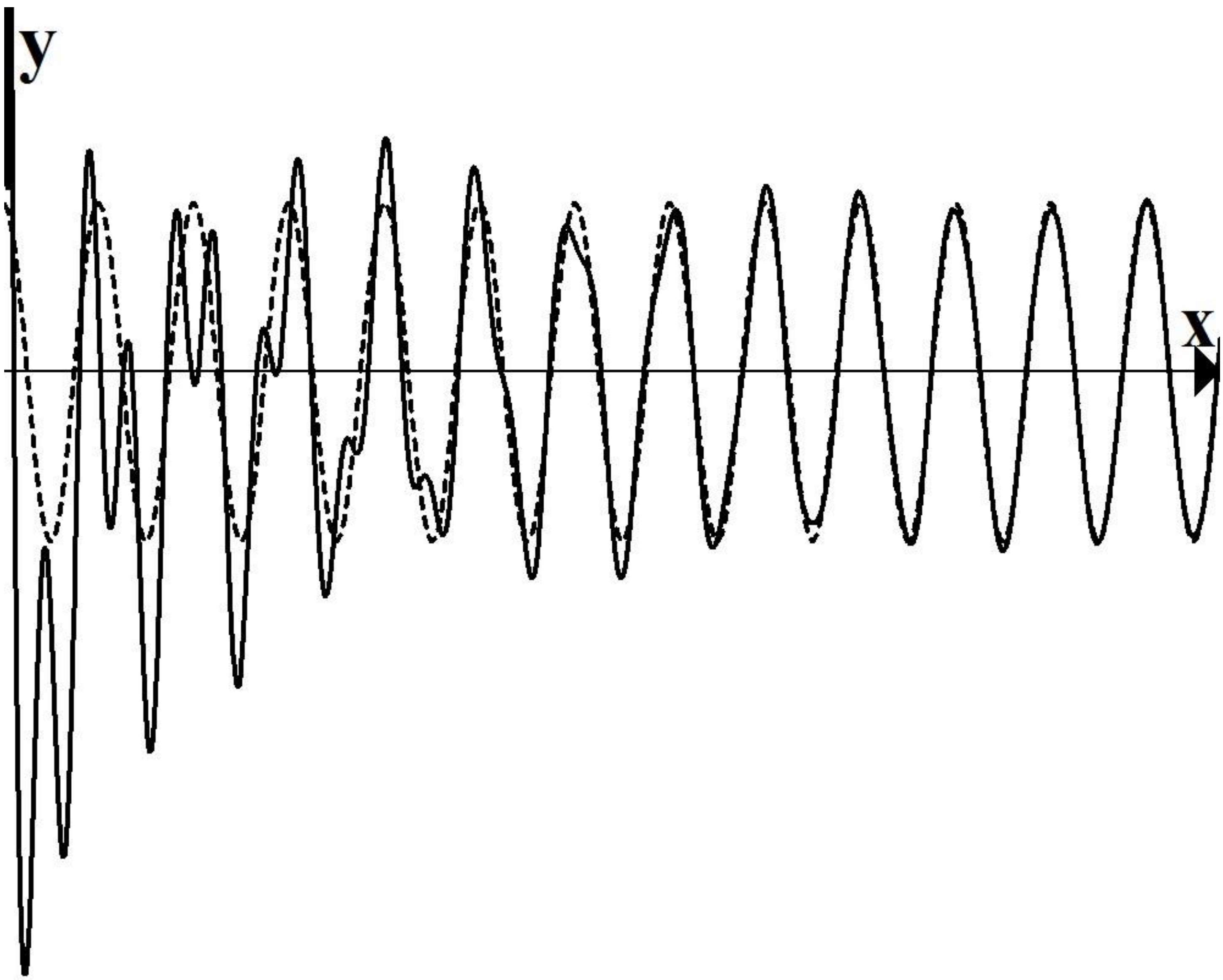


Graphs of sol'ns to

$$u'' + 0.1u' + 5u = 10\cos(t)$$

Steady state solution:

$$U(t) = \frac{40}{16.01}\cos(t) + \frac{1}{16.01}\sin(t)$$

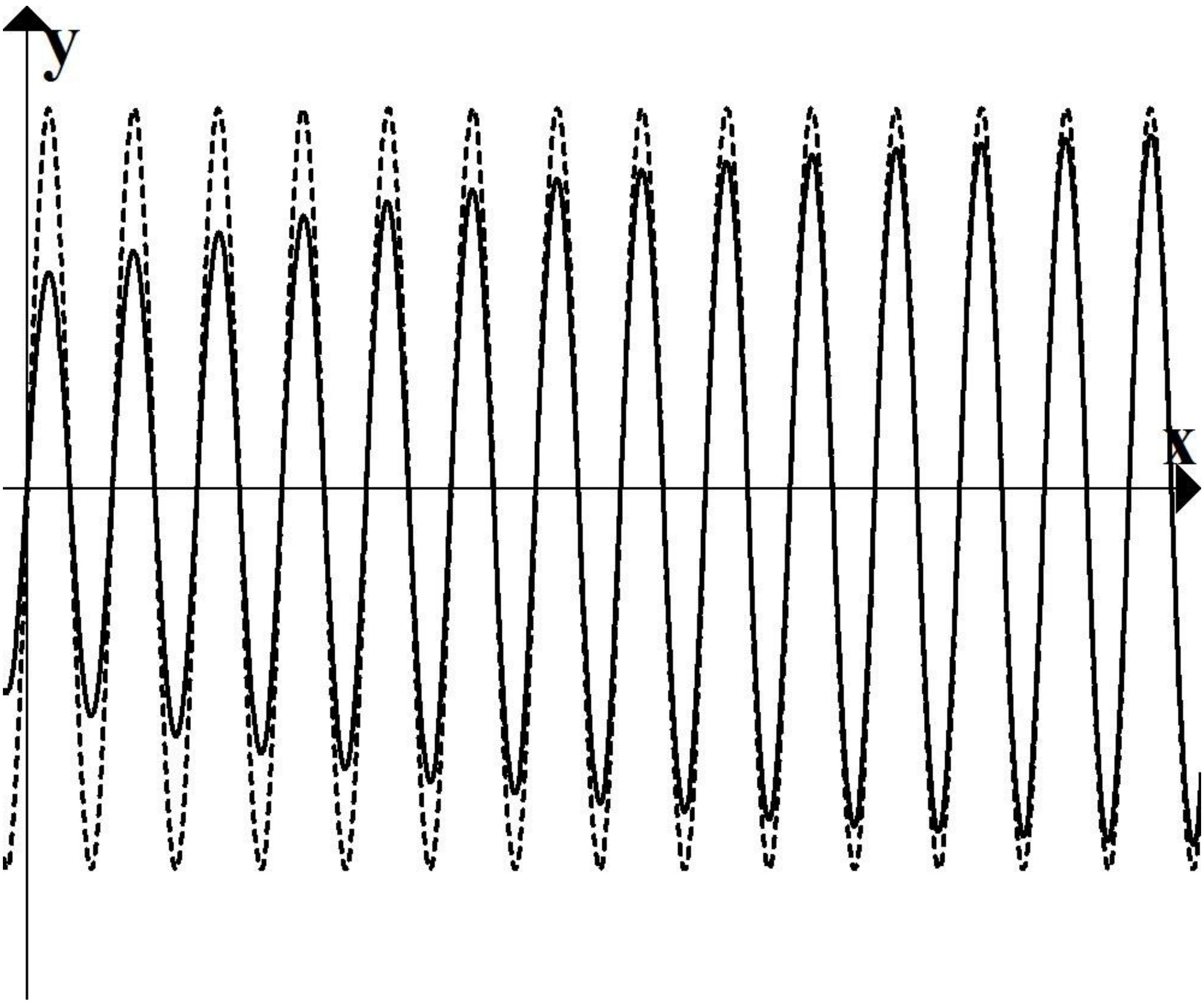


Graphs of sol’ns to

$$u'' + 0.1u' + 5u = 10\cos(\sqrt{5}\,t)$$

Steady state solution:

$$U(t) = \frac{2\sqrt{5}}{0.1}\sin(\sqrt{5}\,t) \approx 44.72\sin(\sqrt{5}\,t)$$



Consider

$$u'' + \gamma u' + 5u = 10\cos(\sqrt{5}\, t)$$

Steady state solution:

$$U(t) = \frac{2\sqrt{5}}{\gamma}\sin(\sqrt{5}\, t)$$

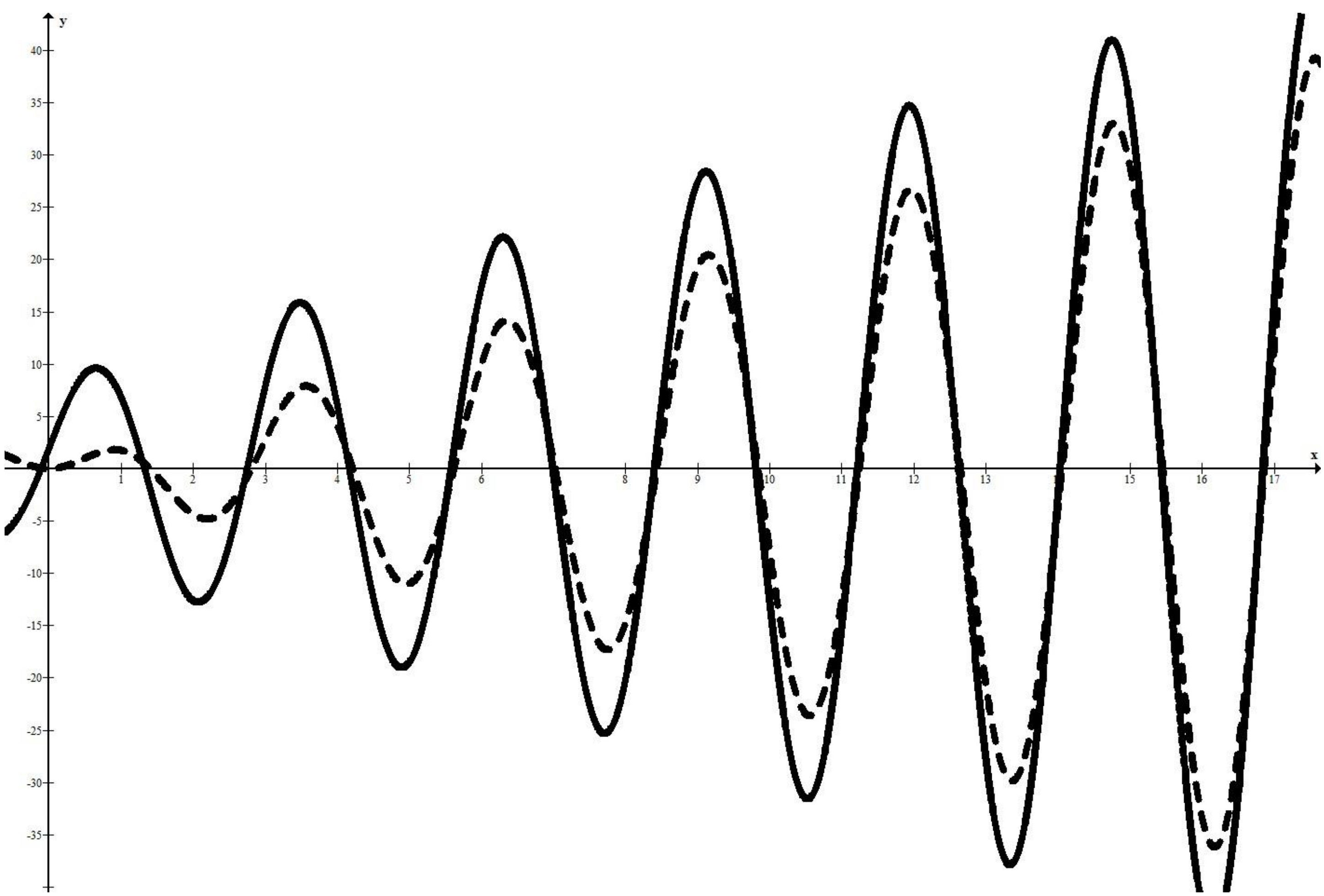
γ	Amplitude of the steady state solution
10	0.447
1	4.47
0.1	44.72
0.01	447.21
0.001	4472.14

If there is NO damping we get:

$$u'' + 5u = 10\cos(\sqrt{5}\,t)$$

General Solution:

$$u(t) = c_1\cos(\sqrt{5}\,t) + c_2\sin(\sqrt{5}\,t) + \sqrt{5}t\sin(\sqrt{5}\,t)$$



General Discussion:

$$mu'' + \gamma u' + ku = F_0 \cos(\omega t)$$

Note: $\omega_0 = \sqrt{\frac{k}{m}}$, $\mu = \sqrt{\frac{k}{m} - \frac{\gamma^2}{4m^2}}$, $\lambda = -\frac{\gamma}{2m}$

Particular Solution:

$$U(t) = A\cos(\omega t) + B\sin(\omega t)$$

Leads to:

$$-\gamma\omega A + (k - m\omega^2)B = 0$$

$$(k - m\omega^2)A + \gamma\omega B = \mathrm{F}_0$$

$$R = \sqrt{A^2 + B^2} = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + \gamma^2\omega^2}}$$

If $\omega \approx \omega_0$, then $R \approx \frac{F_0}{\gamma\omega}$. (Resonance)

For small values of γ, this will be the maximum amplitude.

Aside: $\omega_{max} = \sqrt{\frac{k}{m} - \frac{\gamma^2}{2m^2}}$