Math 307 Section F
Spring 2013
Final Exam
May 22, 2013
Time Limit: 2 Hours

Name (Print): ______________________________

Student ID: ______________________________

This exam contains 12 pages (including this cover page) and 10 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a single, handwritten, one-sided notesheet and a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. Solve the following initial value problems

   (a) (5 points) Solve the following initial value problems:

   $$\frac{dy}{dx} = e^x y - x^2 y, \quad y(0) = 1$$

   (b) (5 points)

   $$y' = 3y + e^t, \quad y(0) = 1/2$$

2. **Propose a Solution Section!**

Directions: The "Propose a Solution" section consists of five linear nonhomogeneous equations. For each of these equations, write down the type of function y (with undetermined coefficients) you would try, in order to get a particular solution. *You do NOT need to solve the equations* For example, if the equation were

$$y'' + 2y' + y = e^t,$$

a *correct answer* would be

$$y = Ae^t,$$

and *incorrect answers* would include

$$y = (At + B)e^t, \quad y = At^2e^{2t}, \quad y = Ae^{3t}, \quad y = A\pi^t$$

Each part is worth 2pts:

(a) (2 points)

$$y'' + 3y' + 2y = t^5e^{4t}$$

(b) (2 points)

$$y'' + 3y' + 2y = (t + 1)e^{-2t}$$

(c) (2 points)

$$y'' - 2y' + y = 2te^t$$

(d) (2 points)

$$y'' + 2y' + y = 4t^2e^t$$

(e) (2 points)

$$y'' - 2y' = te^{2t} - 7e^{2t}$$

3. (10 points) Solve the equation

$$(3xy + y^2)dx + (x^2 + xy)dy = 0$$

by finding an integrating factor.

4. (10 points) Suppose a 120 gallon well-mixed tank initially contains 90 lb. of salt mixed with 90 gal. of water. Salt water (with a concentration of 2 lb/gal) comes into the tank at a rate of 4 gal/min. The solution flows out of the tank at a rate of 3 gal/min. How much salt is in the tank when it is full?

5. (a) (4 points) Find a particular solution of the equation

$$y'' + 2y' + 3y = \cos(t)$$

(b) (2 points) Find a particular solution of the equation

$$y'' + 2y' + 3y = \sin(t)$$

(c) (4 points) Find the general solution of the equation

$$y'' + 2y' + 3y = 3\cos(t) - 2\sin(t)$$

6. (10 points) A mass weighing 2 lb stretches a spring 16 ft. Suppose the mass is displaced an additional 1 ft downward and then released with an upward velocity of 2 ft/s. The mass is in a medium that exerts a viscous resistance of 2 lb when the mass has a velocity of 16 ft/s. Determine the quasi-amplitude and quasifrequency of the resultant motion.

7. (10 points) Find the Laplace transform of

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < 2\pi \\ \cos(t) + t & \text{if } 2\pi \leq t \end{cases}$$

8. (10 points) Find the inverse Laplace transform of

$$F(s) = \frac{3s - 7}{(s + 2)(s^2 + 2s + 3)}$$

9. (10 points) Use Laplace transforms to find the solution to the initial value problem

$$y'' - 9y = e^t \sin(2t), \; y(0) = 1, \; y'(0) = -2.$$

10. (10 points) Use Laplace transforms to find the solution to the initial value problem

$$y'' + 5y' + 6y = f(t), \quad y(0) = 0, \ y'(0) = 0,$$

where

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < \pi \\ 1 & \text{if } \pi \leq t < 2\pi \\ 0 & \text{if } 2\pi \leq t \end{cases}$$

Figure 1: Elementary Laplace Transforms:

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$1/s$
e^{at}	$\frac{1}{s-a}$
t^n, $n \geq 0$ integer	$\frac{n!}{s^{n+1}}$
t^p, $p \geq 0$ real	$\frac{\Gamma(p+1)}{s^{p+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$t^n e^{at}$ $n \geq 0$ integer	$\frac{n!}{(s-a)^{n+1}}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{ct}f(t)$	$F(s-c)$
$f(ct)$	$\frac{1}{c}F(s/c)$
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$
$\delta(t-c)$	e^{-cs}
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
$(-t)^n f(t)$	$F^{(n)}(s)$