

MATH 307D
Final Exam
August 23, 2013

Name _____

Student ID # _____

- Your exam should consist of this cover sheet, followed by 5 problems. Check that you have a complete exam.
- Unless otherwise indicated, show all your work and justify your answers.
- Unless otherwise indicated, your answers should be exact values rather than decimal approximations. For example, $\frac{\pi}{4}$ is an exact answer and is preferable to 0.7854.
- You may use a scientific calculator and one double-sided 8.5×11-inch sheet of handwritten notes. All other electronic devices, including graphing or programmable calculators, and calculators which can do calculus, are forbidden.
- The use of headphones, earbuds during the exam is not permitted. Turn off all your electronic devices and put them away.
- If you need more space, write on the back and indicate this. If you still need more space, raise your hand and I'll give you some extra paper to staple onto the back of your test.
- Academic misconduct will guarantee a score of zero on this exam. **DO NOT CHEAT.**

| Problem | Points | S C O R E |
|---------|--------|-----------|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total: | 50 | |

1. (10 points) Solve the initial value problem

$$y'' - 6y' + 9y = t, \quad y(0) = 0, \quad y'(0) = 0.$$

2. (10 points) Find the general solution to the equation

$$t^2 y'' + 3ty' + y = t.$$

Use the fact that $y = t^{-1}$ is a solution to the corresponding homogeneous equation.

3. (10 points) Using the formula for $L[f''](s)$, find the Laplace transform of $\cos t$.

4. (10 points) Find the general solution to the equation

$$y' = \frac{y^4 - 2y^3 + y^2}{2y - 1}.$$

5. (10 points) Assume we have a lake with volume 1, into which a factory dumps some cyanide, with rate 1 per year. There is a stream going out of the lake, with rate 1 per year. The inflow and outflow have the same rate, but the outflow contains mixed water, and the inflow contains the cyanide with concentration $a(1 + \cos \omega t)$, where $0 < a < 1/2$, $\omega > 0$. Initially, the lake is clean. What is the maximal concentration of cyanide?