Math 300 Assignment 4

PROBLEMS: 3.41, 4.6, 4.8, 4.10, 4.11, 4.12, 4.24, 4.25, 4.26, 4.31, 4.34, 4.37

PLEASE, PLEASE, PLEASE read the special instructions and hints on the back of this sheet before attempting the problems.

ADDITIONAL REQUIRED PROBLEM

PROBLEM I: Prove that if $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are both decreasing, then $h = g \circ f$ is increasing. (this is not a typo!)

PROBLEM II: Put an 'X' (or 'T') in each entry in the table that is TRUE (No proofs required). Assume the domains and targets for the functions labeled with f, g, h_1 and h_2 are as follows:
$f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} - \{0\} \to \mathbb{R}$, $h_1 : \left(-\frac{pi}{2}, \frac{pi}{2}\right) \to \mathbb{R}$, and $h_2 : \mathbb{R} - \{a : a = \frac{k\pi}{2}, k \in \mathbb{Z}\} \to \mathbb{R}$.
Note that the domains and targets given will effect whether the function is injective, surjective, or monotone.

FUNCTION	INJECTIVE?	SURJECTIVE?	BIJECTIVE?	MONOTONE?
$f(x) = 5$				
$f(x) = 3x + 1$				
$f(x) = x^2$				
$f(x) = x^3$				
$f(x) = x^3 - x$				
$f(x) = e^x$				
$f(x) = \sin(x)$				
$f(x) = \arctan(x)$				
$g(x) = 1/x$				
$h_1(x) = \tan(x)$				
$h_2(x) = \tan(x)$				

I hope that this table helps you better understand these terms and helps you come up with counterexamples on the rest of the homework.

The problems above are DUE FRIDAY, FEBRUARY 11th at lecture or during office hours.

HOMEWORK NOTES/HINTS

- PROBLEM 3.41: Part (a) should be quick (at most three lines). That is, don't use induction just plug in particular numerical values of x and y to deduce the result (which is okay because the result is true for all x and y). But induction must be used on part (b).

- PROBLEM 4.10: Prove that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = ax + b$ with $a \neq 0$ is injective and surjective (don't give the proof for $g(x)$ because it would be identical). Then explain why $h(x) = g(f(x)) - f(g(x))$ would not be injective and would not be surjective from $\mathbb{R}$ to $\mathbb{R}$.

- PROBLEM 4.12: Only part (d) is true (use a contrapositive or contradiction proof). For the rest find counterexamples (Note: part (c) is challenging).

- PROBLEM 4.31: You are trying to prove that $y_1 < y_2$ implies $f^{-1}(y_1) < f^{-1}(y_2)$. I suggest you label $x_1 = f^{-1}(y_1)$ and $x_2 = f^{-1}(y_2)$ (then try to prove this implication using the contradiction method).

- PROBLEM 4.34: Try examples involving simple small sets such as $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Try different functions f and g between these sets (or similar small sets). Other examples can often be found, but start small.

- If you finish the homework early or if you are looking for some extra practice try the following problems:

  CHALLENGE PROBLEMS: 3.34, 3.65, 4.13

  These are not due, but I will award at least 1 point of extra credit per challenge problem correctly completed.