Math 300 Assignment 2

PROBLEMS: 1.50, 2.4, 2.10abcd, 2.21, 2.24, 2.26, 2.30, 2.35, 2.38, 2.44c, 2.50b

READ MY DIRECTIONS FOR 2.38 AND 2.44c ON THE BACK OF THIS PAGE
(Ignore the book directions).

DR. LOVELESS PROBLEMS

PROBLEM I: Using a truth table, show that $\neg(P \Rightarrow Q)$ is equivalent to $P \wedge \neg Q$.
(Aside: This is the logical equivalence that we are using when we write a proof by contradiction.)

PROBLEM II: Consider the statement: $P \Rightarrow Q$: "If Bob buys gas, then he will have no money." Now consider the negation $\neg(P \Rightarrow Q)$. A common error that students make is the following: *incorrect negation* "If Bob buys gas, then he will have some money." This is $P \Rightarrow \neg Q$, which is not the negation. This is not the negation because it still gives a true value when Bob does not buy gas. (If Bob does not buy gas, then this statement is still vacuously true). Give the correct negation of the original statement.

PROBLEM III: Find the Summer 2008 (Dr. Loveless) Exam 1 in the Exam Archive on my website (a link is on the left of the website or directly via www.math.washington.edu/~aloveles/Math300Winter2011/examarchive.html). See how many of the problems you can do without looking at the solutions. Write up the solutions to problems 1 and 2 and hand them in with your homework (try to write them up before you look at the solutions).

PROBLEM IV: Many results in mathematics are obtained by taking a known proof and adjusting it to prove a more general or significant result. This exercise gives an example of such an instance: Recall from Assignment 1 (Dr. Loveless Problem IV) that if x and y are nonnegative and $xy \geq 1$, then $(1+x)(1+y) \geq C$ with $C = 4$. Notice that $C = 4$ is *best possible*, that is four is the largest number that will always work because equality can be obtained (by the example $x = 1$, $y = 1$ which makes the product equal to 4). By making appropriate changes to my proof of Assignment 1 Problem IV, see if you can find the C below and then see if you can find a pattern for $C(z)$ (You know that $C(1) = 4$, and $C(2) =$ 'your answer from part 1 below'). That is:

1. Find the largest possible number C such that if x and y are nonnegative and $xy \geq 2$, then $(1+x)(1+y) \geq C$.

2. Generalize the previous part by finding a formula $C(z)$ for the largest possible number C such that if x and y are nonnegative and $xy \geq z$, then $(1+x)(1+y) \geq C(z)$.

The problems above are DUE FRIDAY, JANUARY 21 at lecture or during office hours.

HOMEWORK NOTES/HINTS

- Problem 1.50a should read '$f(C \cap D) \subseteq f(C) \cap f(D)$' (some early printings of the book have the typo $\cup$ instead of $\cap$).
- Problem 2.30: This problem is intended to help you understand the truth table cases of an implication and an if and only if. The statement in part (a) is "If vowel on one side, then there is an odd number on the other." Use common sense to answer (think about what would be needed in a direct proof and a contrapositive proof).
- Problem 2.35: Give a properly structured 'if and only if' proof. You may assume that $a^2 = b^2$ implies $a = b$ or $a = -b$.
- Problem 2.38: Prove part (a) in both directions using only the definitions of even and odd. Namely,

  n is even means $n = 2k$ for some integer k, and

  n is odd means $n = 2t + 1$ for some integer t.

  A contrapositive proof may help in one direction. Give a counterexample to part (b) and tell me if the statement is true in one direction (no proof needed for part (b)).
- Problem 2.44c: Give a truth table argument.
- If you finish the homework early or if you are looking for some extra practice try the following problems:

  CHALLENGE PROBLEMS: 2.32 (give reasoning), 2.40, prove the Pythagorean Theorem

  These are not due, but if you hand them in with your homework, I will award you up to one extra credit point per problem (if you do this, then please put the challenge problems at the end and clearly label them).