

Math 300 Summer 2010 Exam 1

Name: _____

Student ID Number: _____

1	24	
2	23	
3	18	
4	15	
Total	80	

- USE YOUR TIME WISELY! START EACH PROBLEM BY OUTLINING THE PROOFS/IDEAS ON THE PAGE. THEN SPEND NO MORE THAN 12 MINUTES PER PAGE FILLING IN THE DETAILS (AND THEN MOVE ON TO THE NEXT PAGE).
- I will primarily be giving points for structure and format of proofs. Keep your proofs minimal, but leave the essential structure and steps.
- Put your answers on the exam, there are 4 pages (you may use your own paper for scratch work). If you need more space for your answer use the back of the preceding page and indicate that you have done so.
- If you don't know how to prove something, then you should still let me know everything you know about the problem (that is, give definitions and facts you know about the problem and discuss different techniques to prove the theorem). Also, if you are stuck you should try some examples. Don't leave any question blank, show me what you know!

GOOD LUCK!

1. (a) Give the negation, contrapositive, and converse of the following statement avoiding the word 'not' in your final answers. Then determine which statement is true (no proof required):
 ORIGINAL: "For all $a, b \in \mathbb{Z}$, if $a^2 + b^2$ is odd, then a is even or b is even."

(4 pts) NEGATION:

(3 pts) CONTRAPOSITIVE:

(3 pts) CONVERSE:

(3 pts) CIRCLE **ALL** THAT ARE **TRUE**:

ORIGINAL NEGATION CONTRAPOSITIVE CONVERSE

- (b) (3 pts) Find a counterexample to the following statement:
 For all $x, y, z \in \mathbb{N}$, if $x + y + z = 10$, then $xyz > 10$.

- (c) (6 pts) Fill in the truth table below with the appropriate truth values in all cases.

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg Q \Rightarrow (P \wedge \neg Q)$	$\neg P \vee Q$	$(\neg P \vee Q) \Rightarrow Q$
T	T						
T	F						
F	T						
F	F						

(2 pts) How is the statement $\neg Q \Rightarrow (P \wedge \neg Q)$ related to the statement $(\neg P \vee Q) \Rightarrow Q$?
 Clearly circle **all** that apply.

- i. They are converses of each other.
- ii. They are contrapositives of each other.
- iii. They are the negation of each other.
- iv. They are logically equivalent.

2. (a) (13 pts) Let A , B , C , and D be sets.

By giving a formal subset proof, prove that $[A - (B \cap C)] \cap D \subseteq (A - C) \cup B^c$.

(b) (10 pts) Prove if $f(x)$ and $g(x)$ are bounded, then $(f(x))^2 + 3g(x)$ is bounded.

3. (a) (4 pts) Let $f(x) = \sin(x)$.

Find two specific sets A and B such that $f(A \cap B)$ is not equal to $f(A) \cap f(B)$.

(For your sets, give the sets $f(A \cap B)$ and $f(A) \cap f(B)$ as well).

(b) (14 pts) Using induction on n , prove that for all $n \in \mathbb{N}$ with $n \geq 3$ we have

$$\prod_{i=3}^n \left(1 - \frac{4}{i^2}\right) = \frac{(n+1)(n+2)}{6(n-1)n}.$$

4. (15 pts) Let x , y , b and m be integers such that $y = mx + b$ such that **b is even**. Clearly giving a well structure proof and using the precise definitions of even and odd, prove m is odd and x is odd **if and only if** y is odd. (Use an indirect method for one direction).