Math 300 Proof Writing Guidelines

- Clearly state where your proof starts and ends. Make the proof readable by writing in complete sentences when possible. Use proper punctuation and grammer. The argument should be pretty and the so should the presentation.
- **Write with precision**. If you use a variable, make sure you clarify what it represents (it is a real number, or an integer, or does it satisfy any other properties). Use the correct and full statements when you are quoting a definition or fact.
- **'Thus', 'Ergo', ' Hence', 'Therefore', 'So', and 'Then'**: All of these words indicate that you are inferring some statement from previous steps. These words must be preceded or followed by clear justification for the upcoming statement
- **'Since', 'Because', and 'By'**: All of these words are usually attached to a fact or theorem. If you say "Since BLAH BLAH BLAH ...", you better be sure that you know that "BLAH BLAH BLAH" is something that is true (because if it is false or not yet shown in your proof, then you will be marked down).
- **The Order of your Proof is very important!** At this point in the term, all our proofs will be direct proofs (we will talk about other logically equivalent methods later). You must clearly start with the hypotheses and end with the conclusion. For example if you are proving the theorem "If BLAH, then STUFF", then your proof should start with "Assuming BLAH is true" and should clearly end with "Thus, STUFF is true".
- **Write in the first person plural** This is a somewhat odd convention, but you get used to it. Never say "Next, I substitute $x + 1$ into ...", you should instead say, "Next, we substitute $x + 1$ into ..." (as if you are working with the reader as they read your proof).

In 1945 a famous mathematician and teacher named George Pólya wrote a book entitled "How to Solve It". In it he gives step by step advice about how to solve math problems. He says:

1. UNDERSTAND THE PROBLEM : What are the unknowns? Do you clearly understand all the terms and their definitions?
2. DEVISING A PLAN :
   (a) Can you find an analogous problem and solve that?
   (b) Can you find a more general problem that you can solve?
   (c) Can you solve by looking at a few examples and then trying to generalize what you see?
   (d) Can you slightly change the problem to solve it and then use this to solve your original problem?
   (e) Can you break up your work and solve a subproblem or side problem?
   (f) Can you find a similar problem that has already been solved which you can use to solve yours?
   (g) Can you start with the goal and work backwards to better understand the problem?
3. CARRY OUT THE PLAN : Check each step. Can you see they are correct? Can you prove each step?
4. LOOKING BACK : Examine your solution? Could you have solved it more elegantly? Can you use it to prove more general problems?

For us, we have the added step, that after you finish all of this, then you write up a clean, ordered, well-justified proof giving the full solution.