Math 300 Chapters 3 Review

Basic Tools

1. Sum and Product Notation: $\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$ and $\prod_{i=1}^{n} a_i = a_1 a_2 \cdots a_n$.
2. Pulling out last term: It is a basic consequence of the definition of the notation that:

$$\sum_{i=1}^{k+1} a_i = a_{k+1} + \sum_{i=1}^{k} a_i \quad \text{and} \quad \prod_{i=1}^{k+1} a_i = a_{k+1} \prod_{i=1}^{k} a_i.$$

   This is often a good tool to try in your inductive step of your proof.

3. Combining sums and products: Another basic fact is:

$$\left(\sum_{i=1}^{n} a_i\right) + \left(\sum_{i=1}^{n} b_i\right) = \sum_{i=1}^{n}(a_i + b_i) \quad \text{and} \quad \left(\prod_{i=1}^{n} a_i\right)\left(\prod_{i=1}^{n} b_i\right) = \prod_{i=1}^{n}(a_i b_i).$$

Main Proof Techniques / Proof Templates

1. **Basic Induction**

   Theorem For all $n \in \mathbb{N}$, $P(n)$ is true.

   proof We use induction on n.

   Base Step: For $n = 1$, we show that $P(1)$ is true.

   (This usually involves plugging in one to two sides of a formula to show that they are equal).

   Induction Step: Assume $P(k)$ is true for some $k \geq 1$.

$$\vdots$$

   (Show that $P(k+1)$ is true by somehow breaking up the problem so that you can use $P(k)$. Often, we start with one side of the equation or inequality and work toward the other. **At some point in here you must use the inductive hypothesis and you need to tell me when you did so**.)

$$\vdots$$

   Thus, $P(k + 1)$ is true. ■

2. **Strong Induction**

   Theorem For all $n \in \mathbb{N}$, $P(n)$ is true.

   proof We use strong induction on n.

   Base Step: For $n = 1$, we show that $P(1)$ is true.

   (This usually involves plugging in one to two sides of a formula to show that they are equal, if you are using a formula that involves the previous 2 terms then you need to also show that $P(2)$ is true. Similarly for 3 terms, etc.).

Induction Step: Assume $P(1), P(2), ...,$ and $P(k)$ are all true for some $k \geq 1$.

$$\vdots$$

(Show that $P(k + 1)$ is true by somehow breaking up the problem so that you can use the previous values $P(1), P(2), ...,$ and $P(k)$. **At some point in here you must use the inductive hypothesis and you need to tell me when you did so**.)

$$\vdots$$

Thus, $P(k + 1)$ is true. ■