Math 300 Definitions and Theorems from Chapter 2

Essential Definitions

Let P, Q, and R be mathematical statements.

1. $\forall$ = For all. $\exists$ = There exists. s.t. = such that.
2. A mathematical **statement** is a sentence that is either true or false.
3. $P \wedge Q$ = "P AND Q" : true only when both P and Q are true.
4. $P \vee Q$ = "P OR Q" : true when either P is true or Q is true, or both.
5. $P \Rightarrow Q$ = "if P, then Q" = "P implies Q" : true except when P is true and Q is false.
6. $P \Leftrightarrow Q$ = "P if and only if Q" : means P and Q are logically equivalent (have the same truth values in all cases). To prove a statement of the form $P \Leftrightarrow Q$, you must
   (a) Prove $P \Rightarrow Q$.
   (b) Prove $Q \Rightarrow P$.
7. The statement $Q \Rightarrow P$ is called the **converse** of the statement $P \Rightarrow Q$.
8. A **truth table** summarizes the properties of logical statements by listing all possible cases. Here are the truth tables for the basic logical connectives:

P	Q	$P \Rightarrow Q$	$P \vee Q$	$P \wedge Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	F
F	F	T	F	F

Negations and Logic Rules

1. $\neg(P \Rightarrow Q) \Leftrightarrow P \wedge \neg Q$.
2. $\neg(\forall x)P(x) \Leftrightarrow (\exists x)\neg P(x)$.
3. $\neg(\exists x)P(x) \Leftrightarrow (\forall x)\neg P(x)$.

Rule	**For Logic**	**For Sets**
(de Morgan's Laws)	$\neg(P \wedge Q) = \neg P \vee \neg Q$	$(A \cap B)^c = A^c \cup B^c$
(de Morgan's Laws)	$\neg(P \vee Q) = \neg P \wedge \neg Q$	$(A \cup B)^c = A^c \cap B^c$
(Distributive Laws)	$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(Distributive Laws)	$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Main Proof Techniques / Proof Templates

Here is what each proof technique should look like. That is, these are the templates that you are filling in when you give a proof.

1. **Any Direct Proof** ($P \Rightarrow Q$)

   Theorem P implies Q.

   proof Let P be true.

   $$\vdots$$

   (Here you write out the definitions that appear in P and you try to show using logical deductions that the definitions in Q are satisfied)

   $$\vdots$$

   Thus, Q is true. ■

2. **Contrapositive** ($\neg Q \Rightarrow \neg P$)

   Theorem P implies Q

   proof We prove the contrapositive. Let $\neg Q$ be true.

   $$\vdots$$

   (Here you write out the definitions that appear in $\neg Q$ and you try to show using logical deductions that the definitions in $\neg P$ are satisfied)

   $$\vdots$$

   Thus, $\neg P$ is true. ■

3. **Contradiction** ($\neg(P \wedge \neg Q)$)

   Theorem P implies Q

   proof We assume the negation in order to get a contradiction.

   Let P and $\neg Q$ both be true.

   $$\vdots$$

   (Here you write out the definitions that appear in P and in $\neg Q$ and you try to show using logical deductions that a contradiction arises)

   $$\vdots$$

   Thus, we have arrived at a contradiction ($\rightarrow\leftarrow$). ■