This is intended to give you an idea of the length and difficulty of the first midterm exam. This is not an exhaustive review. You will be expected to understand all concepts covered in class and on homework.

1. Let \( A, B, \) and \( C \) be sets.
   
   (a) Prove that \( (A \cup C) - B \subseteq (A - B) \cup C \).
   
   (b) Give an example of three non-empty sets \( A, B, \) and \( C \) for which \( (A \cup C) - B \neq (A - B) \cup C \).

2. Negate each of the following statements.
   
   (a) For every real number \( x \), there is a real number \( y \) such that \( x + y \in \mathbb{Z} \).
   
   (b) There exists an integer \( x \) such that, for every integer \( y \), \( x > y \Rightarrow \frac{x^2}{y} \in \mathbb{N} \).

3. An integer \( n \) is divisible by 3 if \( n = 3k \) for some \( k \in \mathbb{Z} \). Suppose that \( a \) is divisible by 3. Prove that, if \( b \) is not divisible by 3, then \( a + b \) is not divisible by 3. (HINT: Prove the contrapositive.)

4. Use induction to show that, if \( x \) is a real number such that \( 1 + x > 0 \), then \( (1 + x)^n \geq 1 + nx \) for all \( n \in \mathbb{N} \).

5. Let \( A = \{ x \in \mathbb{R} : x \neq 1 \} \) and define \( f : A \rightarrow \mathbb{R} \) by
   
   \[
   f(x) = \frac{x + 1}{x - 1}.
   
   Is \( f(x) \) injective? surjective? bijective? Justify each of your responses with a proof or counterexample.