1a. The differential equation is $2u'' + 6u' + 29u = 0$. The roots of the characteristic equation are $-3/2 \pm 7i/2$. The general solution is $e^{-3t/2}\left[c_1 \cos(7t/2) + c_2 \sin(7t/2)\right]$. Using the initial condition $u(0) = 1$, we get that $c_1 = 1$. Then using the condition $u'(0) = 0$, we find that $c_2 = 3/7$. So our final solution is

$$\boxed{e^{-3t/2}\left[\cos(7t/2) + (3/7)\sin(7t/2)\right].}$$

1b. If we write they function as a pure cosine function, we have $u = \frac{\sqrt{58}}{7} e^{-3t/2} \cos(7t/2 - \delta)$. Since $1/4\,\text{in} = 1/48\,\text{ft}$, we set $\frac{\sqrt{58}}{7} e^{-3t/2} = 1/48$ and solve for t to get

$$\boxed{t = -\frac{2}{3} \ln\left(\frac{7}{48\sqrt{58}}\right) \approx 2.64\,\text{s.}}$$

2 The solution to the homogeneous version of the equation is $y_h = c_1 e^{5t} + c_2 e^{-2t}$. The particular solution will be of the form $Y = At^2 e^{5t} + Bte^{5t}$. Taking derivatives, we get

	t^2e^{5t}	te^{5t}	e^{5t}
Y''	$25A$	$20A + 25B$	$2A + 10B$
Y'	$5A$	$2A + 5B$	B
Y	A	B	

Then we find $A = -1/14$ and $B = 1/49$, so the general solution is

$$\boxed{y = c_1 e^{5t} + c_2 e^{-2t} - \frac{1}{14} t^2 e^{5t} + \frac{1}{49} te^{5t}.}$$

3 Make a substitution $u = u'$. Then the equation is $tu' - 5u = 0$. This is separable, and we get $u = c_1 t^5$. Integrating, we get

$$y = \int u\,dt = \frac{c_1}{6} t^6 + c_2.$$

As long as c_1 is not zero, this is a nonconstant solution. I would probably pick $\boxed{t^6.}$

4 The particular solution is of the form $Y = A\cos 2t + B \sin 2t$. Since there is damping, the solution to the homogeneous version of the problem has an exponential term, so this Y is not a solution to the homogeneous version. We get a system of equations

$$-2A - 3B = 1$$
$$-3A + 2B = 2$$

The solution is $A = -8/13$ and $B = 1/13$. So the steady state solution is

$$\boxed{-(8/13)\cos 2t + (1/13)\sin 2t.}$$

5a The initial value problem is $mu'' + 49u = 0, u(0) = 0.25, u'(0) = -1$. The solution is $0.25\cos(7t/\sqrt{m}) - (\sqrt{m}/7)\sin(7t/\sqrt{m})$. The amplitude is $\sqrt{(0.25)^2 + (m/49)}$. If we set this equal to 0.5 and solve for m, we get $\boxed{m = 9.19\,\text{kg.}}$

5b The period is $\dfrac{2\pi}{7/\sqrt{m}}$. If we set this equal to 1 and solve for m, we get $\boxed{m = 1.24\,\text{kg.}}$