Math 207 - Spring 2022
Exam 1
April 20, 2022

Name: _____

Student ID Number: _____

- There are 5 pages of questions. Make sure your exam contains all these questions.

- You are allowed to use a scientific calculator (**no graphing calculators and no calculators that have calculus capabilities**) and one **hand-written** 8.5 by 11 inch page of notes.

- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put your final answer in the answer blank provided.** Give exact answers wherever possible, if a decimal is required, give four digits after the decimal.

- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.

- If you run out of room, write on provided scratch page at the back of the exam clearly indicate in the problem that you have done so (if you don't then I won't know to look on the scratch page and you don't get credit).

- Raise your hand if you have a question.

- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam.

- You have 50 minutes to complete the exam. Budget your time wisely.

PAGE 1	10	
PAGE 2	10	
PAGE 3	10	
PAGE 4	10	
PAGE 5	10	
Total	50	

GOOD LUCK!

1. (10 pts) The two questions below are unrelated.

   (a) Find the explicit solution to the initial value problem $\dfrac{dy}{dx} = e^{1-y}\sin(4x)$ with $y(0) = 1$.

   ANSWER: $y(x) =$ _____

   (b) Find the general explicit solution to the initial value problem $t\dfrac{dy}{dt} + (1 + 2t)y = 12$. (You may assume $t > 0$).

   ANSWER: $y(t) =$ _____

2. (10 pts) The two questions below are unrelated.

   (a) Find the value of r and s such that $y(x) = e^x + rx + sx^2$ satisfies the differential equation

   $$y'' - y' = 8x + 10.$$

   ANSWER: $r =$ _____ and $s =$ _____

   (b) Consider $\dfrac{dy}{dt} = y(10 - y)(y - 8)^2$

   i. Find and classify the equilibrium solutions (Show your work and *clearly* label each answer as Stable, Unstable or Semistable).

   ANSWERS: $y(t) =$ _____ Classification: _____

   $y(t) =$ _____ Classification: _____

   $y(t) =$ _____ Classification: _____

   ii. If $y = y(t)$ is a solution to the differential equation $\dfrac{dy}{dt} = y(10 - y)(y - 8)^2$ with initial condition $y(1) = 9$, then what is $\lim\limits_{t \to \infty} y(t)$?

   ANSWERS: If $y(1) = 9$, then $\lim\limits_{t \to \infty} y(t) =$ _____

3. (10 pts) Suppose a small metal object, initially at a temperature of 400 degrees Fahrenheit, is placed in an oven. Let $T(t)$ be the temperature of the metal object at time t minutes in degree Fahrenheit. The oven has just been turned off and its temperature is going down according to the function $T_{oven}(t) = 60 + 240e^{-t}$ degrees F, where t is in minutes. Assume the rate of change of temperature of the metal object is proportional to the difference between the temperature of the object and the temperature of the oven, with proportionality constant $k = 0.25 = \frac{1}{4}$. As you should know, the differential equation is:

$$\frac{dT}{dt} = \frac{1}{4}(60 + 240e^{-t} - T).$$

Solve this differentiation equation for the general explicit solution $T(t)$.
(Don't forget to use the initial condition!)

ANSWERS: $y(t) = $ _____

4. (10 pts) The two questions below are unrelated.

(a) Consider $\frac{dy}{dt} = 2y^2 - 6t$ and $y(0) = 1$.

   Use Euler's method with step-size $h = \frac{1}{2}$ to approximate the value of $y(1)$.

                                                    ANSWER: $y(1) \approx$ _____

(b) Bob currently has \$100,000 invested in an account earning 3% interest per year. Assume the rate of grown due to interest is proportional to the balance of the account with proportionality constant $k = 0.03$. In addition, Bob is spending (withdrawing) K dollars per year from the account. The rate of change of the balance is the difference between the interest and the amount he is withdrawing each year (Hint: This is the same idea as our first test prep!). Let $B(t)$ be the balance of the account after t years.

   i. Write down the differential equation and initial condition for this description (your answer will involve K).

          ANSWER: $\frac{dB}{dt} =$ _____ and $B(0) =$ _____

   ii. For some values of K the balance will decrease (eventually spending all the money), but for other values of K the balance will increase. Describe the values of K for which the balance will increase. You do NOT need to solve, use the analysis ideas we learned in section 2.5. (Your answer will be an inequality involving K.).

                    ANSWER: The balance will increase for _____

5. (10 pts) Set-up the differential equation and initial conditions and put a box around your answer. DO NOT SOLVE!

(a) A 500-liter tank currently contains 100-liters of water in which 20 grams of salt have been dissolved. Saltwater with a concentration of 5 grams/liter is pumped in at 50 liters/minute and the well mixed saltwater solution is pumped out at 20 liters/minute (thus the volume of the tank is increasing by 50-20 = 30 L/min). Write down the differential equation and initial condition for the amount of salt $y = y(t)$ in grams in the vat after t minutes.

ANSWER: $\dfrac{dy}{dt} = $ _____ and $y(0) = $ _____

(b) An ice cube is in the shape of a perfect cube with volume 1000 in³. Label the length, height and width of each side x inches. Assume the ice melts (loses volume) at a rate proportional to the surface area of the entire cube with proportionality constant $k = 0.2$. Write down the differential equation and initial condition for the volume $V = V(t)$ of the ice cube.
Note: The only variable in your differential equation should be V.
(Hint: This is a cube, i.e. it has 6 square sides, this is NOT a sphere... you should be able write down the surface area and volume formulas for a cube, start there if you are stuck).

ANSWER: $\dfrac{dV}{dt} = $ _____ and $V(0) = $ _____

Scratch page 1... use this page for more space if needed, very clearly indicated in the problem that you have additional work on "Scratch Page 1" so I that I know to find it here. Also label your extra work below so I know which problem it goes with.

Scratch page 2... use this page for more space if needed, very clearly indicated in the problem that you have additional work on "Scratch Page 2" so I that I know to find it here. Also label your extra work below so I know which problem it goes with.